Entropic fluctuations in quantum two-time measurement framework

joint work in progress with T.Benoist, L.Bruneau, V.Jakšić, C.-A.Pillet

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Context: classical to quantum statistical mechanics

- Transient and Steady Fluctuation relation
- Two-time measurement statistics

Our proposal

- Results
- Proof

Conclusions

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Plan

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Fluctuation relations (following [JPR11])

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Statistical refinement of thermodynamics second law

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If $\mathbb{P}_t^{S,cl}(s)$ is the law of the random variable Σ_t corresponding to *average entropy production rate* et $\bar{\mathbb{P}}_t^{S,cl}(s) = \mathbb{P}_t^{S,cl}(-s)$.
Under general hypothesis (example: ρ Gibbs, TRI)

$$\frac{\mathrm{d}\bar{\mathbb{P}}_t^{S,cl}}{\mathrm{d}\mathbb{P}_t^{S,cl}}(ts) = \mathrm{e}^{-ts}$$

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$$\frac{\mathrm{d}\bar{\mathbb{P}}_t^{S,cl}}{\mathrm{d}\mathbb{P}_t^{S,cl}}(ts) = \mathrm{e}^{-ts}$$

This is called classical <u>transient</u> (ES) fluctuation relation

Fluctuation relations: classical case

general version, following [JPR11]

Classical transient (ES) fluctuation relation

$$\frac{\mathrm{d}\overline{\mathbb{P}}_t^{S,cl}}{\mathrm{d}\mathbb{P}_t^{S,cl}}(s) = \mathrm{e}^{-ts}.$$

equivalent to

$$e_{\mathrm{tr},t}^{cl}(\alpha) = e_{\mathrm{tr},t}^{cl}(1-\alpha)$$

with

$$e_{\mathrm{tr},t}^{cl}(\alpha) = \frac{1}{t}\log\int e^{-t\alpha s}\mathrm{d}\mathbb{P}_t^{S,cl}(ts) = \frac{1}{t}\log\omega(e^{-\alpha t\Sigma_t})$$

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$e_{\mathrm{tr},+}^{cl}(\alpha) := \lim_{t\to\infty}\frac{1}{t}e_{\mathrm{tr},t}^{cl}(\alpha)$ used in large deviations theory

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$$e^{cl}_{\mathrm{ste},t}(\alpha) := \frac{1}{t} \log \omega_{NESS}(e^{-\alpha t \Sigma_t})$$

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- in general $e^{cl}_{\mathrm{ste},t}(\alpha) \neq e^{cl}_{\mathrm{ste},t}(1-\alpha)$ *for t finite*
- typically under strong ergodic hypotesis and if $\mathcal{M}$ compact $e^{cl}_{\mathrm{ste},+}(\alpha) = e^{cl}_{\mathrm{ste},+}(1-\alpha)$ and $e^{cl}_{\mathrm{ste},+}(\alpha) = e^{cl}_{\mathrm{tr},+}(\alpha)$

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Quantum case ?? Transient case

Attempt 1: "Naive quantization"

Underlying idea :

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The success of TTM come with a price: unexpected phenomena (with no classical countepart) due to the invasive role of measurement [Benoist-P.Raquépas19, Benoist-P.Pautrat 20] and *now*

Two-time measurment statistics

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Full (Counting) Statistics [Lesovik, Levitov '93][Levitov, Lee,Lesovik '96]

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Confined systems: described by $(\mathcal{H}, H, \rho)$ $dim\mathcal{H} < \infty$
Given an observable A: $A = \sum_j a_j P_{a_j}$ where $a_j \in \sigma(A)$ P_{a_j} associated spectral projections

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Protocol:
- $t = 0$, we measure A (outcome a_j)
- evolve for time t
- measure again at time t (outcome a_k)

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$\mathbb{P}_{A,t}(\phi)$= probability of measuring a change in A equal to ϕ.

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Fact/Problem: *the measurement perturbes the state, the initial state reduces to* ρ_{am}

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Quantum steady entropic functional: our proposal

How to quantize steady FR? Difficulties:

- A measurement would destroy the steady state
- steady state exists only in the thermodynamic limit
- non-normality of the steady state (to the initial state)in the thermodynamic limit

Setting: Reservoirs $\mathcal{R}_1, \mathcal{R}_2, \ldots \mathcal{R}_M$
coupled directly or through a small system $\mathcal{S}$, $\dim \mathcal{H}_\mathcal{S} = N$

Notation

Reservoirs $\mathcal{R}_1, \mathcal{R}_2, \ldots \mathcal{R}_M$
$(\mathcal{O}_j, \tau_{\mathcal{R}_j,t}, \omega_{\beta_j})$, where ω_{β_j} is β_j KMS state for τ_t^j

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Free Dynamics: $\tau_t^0(A) := \tau_{\mathcal{S},t} \otimes (\tau_{\mathcal{R}_1,t} \otimes \ldots \tau_{\mathcal{R}_M,t}) =: \tau_{\mathcal{S},t} \otimes \tau_{\mathcal{R},t}$
with generator δ_0

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Perturbed dynamics: $\tau_t(A)$ with generator $\delta_0 = \delta_0 + [-, V]$

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Two time measurement framework (start with finite dimensional approximation $\dim \mathcal{H} = n$)
– start with ω initial state as in the transient case
– perform the first measurment at an unknown very large time T
– let the system evolve for time t
– perform the measurment at an unknown very large time $T + t$

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– perform the measurment at an unknown very large time $T + t$
This defines

$$\mathbb{P}^{(n)}_{T,t}, \quad e_{T,t}(\alpha)^{(n)} = \frac{1}{t} \log \int e^{-t\alpha s} \mathrm{d}\mathbb{P}^{(n)}_{T,t}(s)$$

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Proposition (Thermodynamic limit . (T.Benoist, L.Bruneau, V.Jakšić, C.-A.Pillet 21))

It is possible to rewrite $e_{T,t}(\alpha)^{(n)}$ *in term of algebraic objects that survive the limit. Under standard hypothesis,* $e_{T,t}(\alpha) := \lim_{(n)\to\infty} e_{T,t}(\alpha)^{(n)}$ *is well defined correspond to the same formal expression.*

–exact form $e_{T,t}(\alpha)$ shown in the proof

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Definition

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Theorem (direct coupling, no S, T.Benoist, L.Bruneau, V.Jakšić, A.P., C.-A.Pillet '22?)

Let assume the reservoirs are coupled directly (no S); assume the dynamics $\tau^0_{\mathcal{R},t}$ is ergodic. Then

$$e_{T,t}(\alpha) = e_{0,t}(\alpha) =: e_{\mathrm{tr},t}(\alpha)$$

for all $T \in \mathbb{R}$.

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Consequences:

- $e_{\mathrm{ste},t}(\alpha) = e_{\mathrm{tr},t}(\alpha)$

Theorem (direct coupling, no S, T.Benoist, L.Bruneau, V.Jakšić, A.P., C.-A.Pillet '22?)

Let assume the reservoirs are coupled directly (no S); assume the dynamics $\tau^0_{\mathcal{R},t}$ is ergodic. Then

$$e_{T,t}(\alpha) = e_{0,t}(\alpha) =: e_{\mathrm{tr},t}(\alpha)$$

for all $T \in \mathbb{R}$.

Consequences:

- $e_{\mathrm{ste},t}(\alpha) = e_{\mathrm{tr},t}(\alpha)$
- If the symmetry true for $e_{\mathrm{tr},t}(\alpha)$, then for the steady functional $e_{\mathrm{ste},t}(\alpha)$ also satisfy the symmetry AT FINITE TIME t;
- if $e_{\mathrm{ste},+}(\alpha)$, $e_{0,+}(\alpha)$ exists, they are equal.

Theorem (coupling through $\mathcal{S}$, T.Benoist, L.Bruneau, V.Jakšić, A.P., C.-A.Pillet 22?)

Let assume the reservoirs are through a small system $\mathcal{S}$; assume the dynamics $\tau^0_{\mathcal{R},t}$ is ergodic. Then

$$e_{\mathrm{ste},+}(\alpha) = e_{0,+}(\alpha) =: e_{\mathrm{tr},+}(\alpha)$$

Remark

In both theorems:

1. *No additional hypothesis on the perturbed dynamics*
2. *Underlying mechanism: invasive measurment role*
3. *General proof, with algebraic methods (no need for resonance analysis model by model)*
4. *Need for thermodynamic limit*

Hint of the proofs

Consider the GNS representation associated to ω; $(\mathcal{H}_\omega, \pi_\omega, \Omega)$

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Liouvillean: any operator such that $\pi_\omega(\tau_t(A)) = e^{itL}\pi_\omega(A)e^{-itL}$
not uniquely defined.
L_∞ such that $L_\infty \Omega = \Omega$
L_α deformed Liovillean
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Direct coupling: If the dynamics on $\mathcal{R}$ is ergodic, 0 is a simple eigenvalue for L_0 and $1_{\{0\}}(L_0) = |\Omega\rangle\langle\Omega|$

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Coupling through a small system $\mathcal{S}$: If the dynamics on $\mathcal{R}$ is ergodic, 0 is a simple eigenvalue for L_0 and $\ker(L) = \ker(L_\mathcal{S}) \otimes \Omega_\mathcal{R}$; equality is attained in in the long time limit $t \to \infty$

Conclusions

- we have introduced a proposal for quantum steady (GC) entropic functional $e_{\mathrm{ste},+}(\alpha)$
- we have shown $e_{\mathrm{ste},+}(\alpha) = e_{\mathrm{tr},+}(\alpha)$ under very weak ergodicity hypothesis
- direct measurement on infinitely extended reservoir is an idealization. We are able to write similar (less general) results ins the framework of an "ancilla" measurement using resonance theory .

Thank you for your attention !