Quasiparticles - Holes in the Fermi Sea

Peter Pickl

Mathematisches Institut
LMU München

20. August 2019

Question

- Gas of many Fermions in the ground state
  i.e. filled Fermi-sea
- Excite one of the particles
- The hole behaves like a particle itself
  it propagates like a particle with opposite charge
  interacts like a particle with opposite charge

Question

- Gas of many Fermions in the ground state
  i.e. filled Fermi-sea
- Excite one of the particles
- The hole behaves like a particle itself
  it propagates like a particle with opposite charge
  interacts like a particle with opposite charge

Question

- Gas of many Fermions in the ground state
  i.e. filled Fermi-sea
- Excite one of the particles
- The hole behaves like a particle itself
  it propagates like a particle with opposite charge
  interacts like a particle with opposite charge

Question

- Gas of many Fermions in the ground state
  i.e. filled Fermi-sea
- Excite one of the particles
- The hole behaves like a particle itself
  it propagates like a particle with opposite charge
  interacts like a particle with opposite charge

Question

- Gas of many Fermions in the ground state
  i.e. filled Fermi-sea
- Excite one of the particles
- The hole behaves like a particle itself
  it propagates like a particle with opposite charge
  interacts like a particle with opposite charge

Question

- Gas of many Fermions in the ground state
  i.e. filled Fermi-sea
- Excite one of the particles
- The hole behaves like a particle itself
  it propagates like a particle with opposite charge
  interacts like a particle with opposite charge

Interaction with the Fermi sea

- Model: Tracer particle interacting with the Fermi sea, Torus in d dimensions

$$H = -\Delta_y + \sum_{j=1}^{N} -\Delta_{x_j} + V(x_j - y)$$

$$\Psi_0 = \Psi^{GS} \otimes \phi$$

- Bosonic case: Friction and diffusion
- Theorem: Tracer particle moves like a free particle for $d = 1, 2$
  i.e. $\mu^{\Psi} \to | \phi^{free} \rangle \langle \phi^{free} |$ in the high density limit
  $d = 2$: joint work with M. Jeblick, D. Mitrouskas and S. Petrat (CMP 2017)

Interaction with the Fermi sea

- Model: Tracer particle interacting with the Fermi sea, Torus in d dimensions

$$H = -\Delta_y + \sum_{j=1}^{N} -\Delta_{x_j} + V(x_j - y)$$

$$\Psi_0 = \Psi^{GS} \otimes \phi$$

- Bosonic case: Friction and diffusion
- Theorem: Tracer particle moves like a free particle for $d = 1, 2$
  i.e. $\mu^{\Psi} \to | \phi^{free} \rangle \langle \phi^{free} |$ in the high density limit
  $d = 2$: joint work with M. Jeblick, D. Mitrouskas and S. Petrat (CMP 2017)

Interaction with the Fermi sea

- Model: Tracer particle interacting with the Fermi sea, Torus in d dimensions

$$H = -\Delta_y + \sum_{j=1}^{N} -\Delta_{x_j} + V(x_j - y)$$

$$\Psi_0 = \Psi^{GS} \otimes \phi$$

- Bosonic case: Friction and diffusion
- Theorem: Tracer particle moves like a free particle for $d = 1, 2$ i.e. $\mu^{\Psi} \to |\phi^{free}\rangle\langle\phi^{free}|$ in the high density limit
  $d = 2$: joint work with M. Jeblick, D. Mitrouskas and S. Petrat (CMP 2017)

Interaction with the Fermi sea

- Model: Tracer particle interacting with the Fermi sea, Torus in d dimensions

$$H = -\Delta_y + \sum_{j=1}^{N} -\Delta_{x_j} + V(x_j - y)$$

$$\Psi_0 = \Psi^{GS} \otimes \phi$$

- Bosonic case: Friction and diffusion
- Theorem: Tracer particle moves like a free particle for $d = 1, 2$ i.e. $\mu^{\Psi} \to |\phi^{free}\rangle\langle\phi^{free}|$ in the high density limit
  $d = 2$: joint work with M. Jeblick, D. Mitrouskas and S. Petrat (CMP 2017)

Interaction with the Fermi sea

- Model: Tracer particle interacting with the Fermi sea, Torus in d dimensions

$$H = -\Delta_y + \sum_{j=1}^{N} -\Delta_{x_j} + V(x_j - y)$$

$$\Psi_0 = \Psi^{GS} \otimes \phi$$

- Bosonic case: Friction and diffusion
- Theorem: Tracer particle moves like a free particle for $d = 1, 2$ i.e. $\mu^{\Psi} \to |\phi^{free}\rangle\langle\phi^{free}|$ in the high density limit
  $d = 2$: joint work with M. Jeblick, D. Mitrouskas and S. Petrat (CMP 2017)

Interaction with the Fermi sea

- Model: Tracer particle interacting with the Fermi sea, Torus in d dimensions

$$H = -\Delta_y + \sum_{j=1}^{N} -\Delta_{x_j} + V(x_j - y)$$

$$\Psi_0 = \Psi^{GS} \otimes \phi$$

- Bosonic case: Friction and diffusion
- Theorem: Tracer particle moves like a free particle for $d = 1, 2$
  i.e. $\mu^{\Psi} \to | \phi^{free} \rangle \langle \phi^{free} |$ in the high density limit
  $d = 2$: joint work with M. Jeblick, D. Mitrouskas and S. Petrat (CMP 2017)

Interaction with the Fermi sea

- Model: Tracer particle interacting with the Fermi sea, Torus in d dimensions

$$H = -\Delta_y + \sum_{j=1}^{N} -\Delta_{x_j} + V(x_j - y)$$

$$\Psi_0 = \Psi^{GS} \otimes \phi$$

- Bosonic case: Friction and diffusion
- Theorem: Tracer particle moves like a free particle for $d = 1, 2$ i.e. $\mu^{\Psi} \rightarrow | \phi^{free} \rangle \langle \phi^{free} |$ in the high density limit
  $d = 2$: joint work with M. Jeblick, D. Mitrouskas and S. Petrat (CMP 2017)

Interaction with the Fermi sea

- Model: Tracer particle interacting with the Fermi sea, Torus in d dimensions

$$H = -\Delta_y + \sum_{j=1}^{N} -\Delta_{x_j} + V(x_j - y)$$

$$\Psi_0 = \Psi^{GS} \otimes \phi$$

- Bosonic case: Friction and diffusion
- Theorem: Tracer particle moves like a free particle for $d = 1, 2$ i.e. $\mu^{\Psi} \rightarrow | \phi^{free} \rangle \langle \phi^{free} |$ in the high density limit
  $d = 2$: joint work with M. Jeblick, D. Mitrouskas and S. Petrat (CMP 2017)

Heuristic of the proof

- Antisymmetry: Fluctuation of particles is suppressed
- Example: $\chi(x_1, x_2) = e^{ikx_1} \pm e^{ikx_2}$
- Bosons cluster, Fermions avoid each other
- ρ large: Bosons Var $\sim \rho$; Fermions Var $\sim \rho^{1-d^{-1}}$ ($\ln \rho$ if $d = 1$)
- Fluctuation still large! However tracer still moves freely!
- Main contribution from particles with high momentum.
  Short time of interaction with tracer.
  Small impulse!

Heuristic of the proof

- Antisymmetry: Fluctuation of particles is suppressed
- Example: $\chi(x_1, x_2) = e^{ikx_1} \pm e^{ikx_2}$
- Bosons cluster, Fermions avoid each other
- ρ large: Bosons Var $\sim \rho$; Fermions Var $\sim \rho^{1-d^{-1}}$ ($\ln \rho$ if $d = 1$)
- Fluctuation still large! However tracer still moves freely!
- Main contribution from particles with high momentum.
  Short time of interaction with tracer.
  Small impulse!

Heuristic of the proof

- Antisymmetry: Fluctuation of particles is suppressed
- Example: $\chi(x_1, x_2) = e^{ikx_1} \pm e^{ikx_2}$
- Bosons cluster, Fermions avoid each other
- ρ large: Bosons Var $\sim \rho$; Fermions Var $\sim \rho^{1-d^{-1}}$ ($\ln \rho$ if $d = 1$)
- Fluctuation still large! However tracer still moves freely!
- Main contribution from particles with high momentum.
  Short time of interaction with tracer.
  Small impulse!

Heuristic of the proof

- Antisymmetry: Fluctuation of particles is suppressed
- Example: $\chi(x_1, x_2) = e^{ikx_1} \pm e^{ikx_2}$
- Bosons cluster, Fermions avoid each other
- ρ large: Bosons Var $\sim \rho$; Fermions Var $\sim \rho^{1-d^{-1}}$ ($\ln \rho$ if $d = 1$)
- Fluctuation still large! However tracer still moves freely!
- Main contribution from particles with high momentum.
  Short time of interaction with tracer.
  Small impulse!

Heuristic of the proof

- Antisymmetry: Fluctuation of particles is suppressed
- Example: $\chi(x_1, x_2) = e^{ikx_1} \pm e^{ikx_2}$
- Bosons cluster, Fermions avoid each other
- ρ large: Bosons Var $\sim \rho$; Fermions Var $\sim \rho^{1-d^{-1}}$ ($\ln \rho$ if $d = 1$)
- Fluctuation still large! However tracer still moves freely!
- Main contribution from particles with high momentum.
  Short time of interaction with tracer.
  Small impulse!

Heuristic of the proof

- Antisymmetry: Fluctuation of particles is suppressed
- Example: $\chi(x_1, x_2) = e^{ikx_1} \pm e^{ikx_2}$
- Bosons cluster, Fermions avoid each other
- ρ large: Bosons Var $\sim \rho$; Fermions Var $\sim \rho^{1-d^{-1}}$ ($\ln \rho$ if $d = 1$)
- Fluctuation still large! However tracer still moves freely!
- Main contribution from particles with high momentum.
  Short time of interaction with tracer.
  Small impulse!

Heuristic of the proof

- Antisymmetry: Fluctuation of particles is suppressed
- Example: $\chi(x_1, x_2) = e^{ikx_1} \pm e^{ikx_2}$
- Bosons cluster, Fermions avoid each other
- ρ large: Bosons Var $\sim \rho$; Fermions Var $\sim \rho^{1-d^{-1}}$ ($\ln \rho$ if $d = 1$)
- Fluctuation still large! However tracer still moves freely!
- Main contribution from particles with high momentum.
  Short time of interaction with tracer.
  Small impulse!

Heuristic of the proof

- Antisymmetry: Fluctuation of particles is suppressed
- Example: $\chi(x_1, x_2) = e^{ikx_1} \pm e^{ikx_2}$
- Bosons cluster, Fermions avoid each other
- ρ large: Bosons Var $\sim \rho$; Fermions Var $\sim \rho^{1-d^{-1}}$ ($\ln \rho$ if $d = 1$)
- Fluctuation still large! However tracer still moves freely!
- Main contribution from particles with high momentum.
  Short time of interaction with tracer.
  Small impulse!

Heuristic of the proof

- Antisymmetry: Fluctuation of particles is suppressed
- Example: $\chi(x_1, x_2) = e^{ikx_1} \pm e^{ikx_2}$
- Bosons cluster, Fermions avoid each other
- ρ large: Bosons Var $\sim \rho$; Fermions Var $\sim \rho^{1-d^{-1}}$ ($\ln \rho$ if $d = 1$)
- Fluctuation still large! However tracer still moves freely!
- Main contribution from particles with high momentum.
  Short time of interaction with tracer.
  Small impulse!

Outlook

- Consider Fermi sea with hole
  Prove that tracer effectively interacts with an anti-particle
- Consider full interacting model
- Consider $d > 2$

Thank you!

Outlook

- Consider Fermi sea with hole
  Prove that tracer effectively interacts with an anti-particle
- Consider full interacting model
- Consider $d > 2$

Thank you!

Outlook

- Consider Fermi sea with hole
  Prove that tracer effectively interacts with an anti-particle
- Consider full interacting model
- Consider $d > 2$

Thank you!

Outlook

- Consider Fermi sea with hole
  Prove that tracer effectively interacts with an anti-particle
- Consider full interacting model
- Consider $d > 2$

Thank you!

Outlook

- Consider Fermi sea with hole
  Prove that tracer effectively interacts with an anti-particle
- Consider full interacting model
- Consider $d > 2$

Thank you!