Quasiparticles - Holes in the Fermi Sea

Peter Pickl

Mathematisches Institut
LMU München

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Question

- Gas of many Fermions in the ground state
  i.e. filled Fermi-sea
- Excite one of the particles
- The hole behaves like a particle itself
  it propagates like a particle with opposite charge
  interacts like a particle with opposite charge
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Interaction with the Fermi sea

- Model: Tracer particle interacting with the Fermi sea, Torus in \( d \) dimensions
  \[
  H = -\Delta_y + \sum_{j=1}^{N} -\Delta_{x_j} + V(x_j - y)
  \]
  \[
  \Psi_0 = \psi^{GS} \otimes \phi
  \]

- Bosonic case: Friction and diffusion

- Theorem: Tracer particle moves like a free particle for \( d = 1, 2 \)
  i.e. \( \mu^\Psi \to |\phi^{free}\rangle \langle \phi^{free}| \) in the high density limit
  \( d = 2 \): joint work with M. Jeblick, D. Mitrouskas and S. Petrat
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Heuristic of the proof

- Antisymmetry: Fluctuation of particles is suppressed
- Example: $\chi(x_1, x_2) = e^{ikx_1} \pm e^{ikx_2}$
- Bosons cluster, Fermions avoid each other
- $\rho$ large: Bosons Var $\sim \rho$; Fermions Var $\sim \rho^{1-d^{-1}} \ln \rho$ if $d = 1$
- Fluctuation still large! However tracer still moves freely!
- Main contribution from particles with high momentum.
  Short time of interaction with tracer.
  Small impulse!
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Outlook

- Consider Fermi sea with hole
  Prove that tracer effectively interacts with an anti-particle
- Consider full interacting model
- Consider $d > 2$

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