Heat fluctuations in the two-time measurement framework and ultraviolet regularity

joint work with T. Benoist, (Y. Pautrat) and R. Raquépas

Annalisa Panati,
CPT, Université de Toulon
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Context: quantum statistical mechanics
Fluctuation relations

Two-time measurement statistics
Conservation laws

Heat fluctuations: theorems and results
Bounded perturbations

Conclusions
Context: quantum statistical mechanics

Fluctuation relations

Plan

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Fluctuation relations

Classical case: [Evans-Cohen-Morris '93] numerical experiences
[Evans-Searls '94] [Gallavotti Cohen '94] theoretical explanation

Statistical refinement of thermodynamics second law
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work in driven system [Bochkov-Kuzovlev '70s] [Jaryzinski '97] [Crooks '99] etc
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**Quantum case:** ??
Quantization of fluctuation relations

Quantum case:

**Attempt 1: "Naive quantization"**

**Underlying idea**: define an observable \( \Sigma_t = \frac{1}{t} (S_t - S) \) on \( \mathcal{H} \) and consider the spectral measure \( \mu \Sigma_t \)

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**Underlying idea**: define an observable $\Sigma_t = \frac{1}{t} (S_t - S)$ on $\mathcal{H}$ and consider the spectral measure $\mu_{\Sigma_t}$

—attempted in work related literature [Bochkov-Kuzovlev ’70s–’80s])

—attempted in the ’90, called "naive quantization"
Quantization of fluctuation relations

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leads to **NO-fluctuation relations**!!!!
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**Attempt 2**: Measurement has been neglected. Associate to \( S \) the \textit{two-time measurement statistics} \( \mathbb{P}_t^S \) defined as difference between two measurement
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leads to **fluctuation relations**

At the level of averages and variances, there is no difference!
Two-time measurement statistics

Full (Counting) Statistics [Lesovik, Levitov ’93][Levitov, Lee,Lesovik ’96]
Two-time measurement statistics

Full (Counting) Statistics [Lesovik, Levitov ’93][Levitov, Lee, Lesovik ’96]

Confined systems: described by \((\mathcal{H}, H, \rho)\) \(\dim \mathcal{H} < \infty\)

Given an observable \(A\):

\[ A = \sum_j a_j P_{a_j} \text{ where } a_j \in \sigma(A) P_{a_j} \]

associated spectral projections
Two-time measurement statistics

Full (Counting) Statistics [Lesovik, Levitov '93][Levitov, Lee, Lesovik '96]

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Procedure:
- \(t = 0\), we measure \(A\) (outcome \(a_j\))
- evolve for time \(t\)
- measure again at time \(t\) (outcome \(a_k\))
Full (Counting) Statistics [Lesovik, Levitov ’93][Levitov, Lee, Lesovik ’96]

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Two-time measurement distribution of \(A\):
\(P_{A,t}(\phi) = \) probability of measuring a change in \(A\) equal to \(\phi\).
Two-time measurement statistics

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Heat fluctuations in the two-time measurement framework and ultraviolet regularity

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Two-time measurement statistics

Procedure:
- At time $t = 0$, we measure $A$ (outcome $a_j$)
- Evolve for time $t$
- Measure again at time $t$ (outcome $a_k$)

$P_{A,t}(\phi) = \text{probability of measuring a change in } A \text{ equal to } \phi$

In confined system:

$$P_{A,t}(\phi) = \sum_{a_k - a_j = \phi} \text{tr} (\rho P_{a_j} e^{-i t H} P_{a_k})$$

$$= \sum_{a_k - a_j = \phi} \text{tr} (e^{-i t H} P_{a_j} \rho P_{a_j} e^{i t H})$$

with $\rho_{a} = \frac{1}{\text{tr}(\rho P_{a_j})} P_{a_j} \rho P_{a_j}$. 

Fact/Problem: the measurement perturbs the state, the initial state reduces to $\rho_a$.

Remark: $\text{supp}(P_{A,t})$ is included on the set of possible $A$-differences.
Two-time measurement statistics

Procedure:
- $t = 0$, we measure $A$ (outcome $a_j$)
- evolve for time $t$
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\[ P_{A,t}(\phi) = \text{probability of measuring a change in } A \text{ equal to } \phi \]

In confined system:

\[ P_{A,t}(\phi) = \sum_{a_k - a_j = \phi} \text{tr} \left( \rho P_{a_j} \rho P_{a_k} e^{-i t H} \right) = \sum_{a_k - a_j = \phi} \text{tr} \left( e^{-i t H} P_{a_j} \rho P_{a_k} e^{i t H} \right) \]

with

\[ \rho_{am} = \frac{1}{\text{tr} \left( \rho P_{a_j} \right)} P_{a_j} \rho P_{a_j} \]
Two-time measurement statistics

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Two-time measurement statistics

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$P_{A,t}(\phi) =$ probability of measuring a change in $A$ equal to $\phi$

In confined system:

$$P_{A,t}(\phi) = \text{tr}(\rho P_{aj}) \text{tr}(e^{-itH} \rho_{am} e^{itH} P_{ak})$$

with

$$\rho_{am} = \frac{1}{\text{tr}(\rho P_{aj})} P_{aj} \rho P_{aj}.$$  

Fact/Problem: *the measurement perturbs the state, the initial state reduces to $\rho_{am}$*
Two-time measurement statistics

Procedure:
- $t = 0$, we measure $A$ (outcome $a_j$)
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In confined system:

$$P_{A,t}(\phi) = \sum_{a_k-a_j=\phi} \text{tr}(\rho P_{aj}) \text{tr}(e^{-itH} \rho_{am} e^{itH} P_{ak})$$

with

$$\rho_{am} = \frac{1}{\text{tr}(\rho P_{aj})} P_{aj} \rho P_{aj}.$$  

Fact/Problem: the measurement perturbs the state, the initial state reduces to $\rho_{am}$

Remark: supp($P_{A,t}$) is included on the set of possible $A$-differences
**Two-time measurement statistics**

**Procedure:**
- \( t = 0 \), we measure \( A \) (outcome \( a_j \))
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In confined system:

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P_{A,t}(\phi) = \sum_{a_k-a_j=\phi} \text{tr}(\rho P_{aj})\text{tr}(e^{-itH}\rho_{am}e^{itH}P_{ak})
\]

\[
= \sum_{a_k-a_j=\phi} \text{tr}(e^{-itH}P_{aj}\rho P_{aj}e^{itH}P_{ak})
\]

with

\[
\rho_{am} = \frac{1}{\text{tr}(\rho P_{aj})} P_{aj}\rho P_{aj}.
\]

**Fact/Problem:** the measurement perturbs the state, the initial state reduces to \( \rho_{am} \)

**Remark:** \( \text{supp}(P_{A,t}) \) is included on the set of possible \( A \)-differences
Two-time measurement statistics

Conservation laws

---

Given a classical observable $C$ and an initial state $\rho$, we call $C$-statistics the probability measure $P_C$ such that $\int f(s)dP_C(s) = \int f(C)d\rho$ for all $f \in \mathcal{B}(\mathbb{R})$. 

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Two-time measurement statistics

Conservation laws

Classical system: \((\mathcal{M}, H, \rho)\)

\(^1\)Given a classical observable \(C\) and an initial state \(\rho\), we call \(C\)-statistics the probability measure \(\mathbb{P}_C\) such that \(\int f(s)d\mathbb{P}_C(s) = \int f(C)d\rho\) for all \(f \in \mathcal{B}(\mathbb{R})\).
Two-time measurement statistics

Conservation laws

Classical system: \((\mathcal{M}, H, \rho)\)

Two-time measurement statistics is equivalent to the law \(P_{\triangle A_t}\) associated to \(\triangle A_t := A_t - A\). ¹

¹Given a classical observable \(C\) and an initial state \(\rho\), we call \(C\)-statistics the probability measure \(P_C\) such that \(\int f(s) dP_C(s) = \int f(C) d\rho\) for all \(f \in B(\mathbb{R})\)
Two-time measurement statistics

Conservation laws

**Classical system:** $(\mathcal{M}, H, \rho)$

Two-time measurement statistics is equivalent to the law $\mathbb{P}_{\triangle A_t}$ associated to $\triangle A_t := A_t - A$. ¹

Assume $A + B$ is conserved. The identity as classical observables $\triangle A_t = A_t - A = -(B_t - B) = -\triangle B_t$ yields the identity

$$\mathbb{P}_{\triangle A_t} = \mathbb{P}_{-\triangle B_t}$$

¹Given a classical observable $C$ and an intial state $\rho$, we call $C$-statistics the probability measure $\mathbb{P}_C$ such that $\int f(s)d\mathbb{P}_C(s) = \int f(C)d\rho$ for all $f \in \mathcal{B}(\mathbb{R})$.  "Heat fluctuations in the two-time measurement framework and ultraviolet regularity"  

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Classical system: \((\mathcal{M}, H, \rho)\)

Two-time measurement statistics is equivalent to the law \(P_{\Delta A_t}\) associated to \(\Delta A_t := A_t - A\). \(^1\)

Assume \(A + B\) is conserved. The identity as classical observables
\[
\Delta A_t = A_t - A = -(B_t - B) = -\Delta B_t
\]
yields the identity

\[
P_{\Delta A_t} = P_{-\Delta B_t}
\]

Quantum system \(A_t - A = -(B_t - B)\) as operators yields identity
between all spectral measures

\[
\mu_{\Delta A_t} = \mu_{-\Delta B_t} \quad \text{but} \quad P_{A,t} \neq P_{-B,t}
\]

---

\(^1\)Given a classical observable \(C\) and an initial state \(\rho\), we call \(C\)-statistics the probability measure \(P_C\) such that \(\int f(s) dP_C(s) = \int f(C) d\rho\) for all \(f \in \mathcal{B}(\mathbb{R})\).
Two-time measurement statistics

Conservation laws-energy balance

Classical system \((\mathcal{M}, H, \rho)\)

\(H = H_0 + V\) \(\rho\) is invariant for the dynamics associated to \(H_0\)

\((H_0\) interpreted as heat)
Two-time measurement statistics

Conservation laws—energy balance

**Classical system** $(\mathcal{M}, H, \rho)$

$H = H_0 + V$ \quad \rho \text{ is invariant for the dynamics associated to } H_0 \\
(H_0 \text{ interpreted as heat})

**Energy conservation:**

$\triangle H_{0,t} = H_{0,t} - H_0 = -(V_t - V) = (-V)_t - (-V)$ as function on $\mathcal{M}$
Two-time measurement statistics

Conservation laws - energy balance

Classical system \((\mathcal{M}, H, \rho)\)

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\((H_0\) interpreted as heat\)

Energy conservation:

\[ \triangle H_{0,t} = H_{0,t} - H_0 = -(V_t - V) = (-V)_t - (-V) \]

as function on \(\mathcal{M}\)

In particular if \(V\) is bounded by \(C\) then

\[ \sup_t |\triangle H_{0,t}| < 2C \]

and \(\text{supp}(\mathbb{P}_{\triangle H_{0,t}})\) bounded
Two-time measurement statistics

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Quantum system \((\mathcal{H}, H, \rho)\)

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Two-time measurement statistics

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Quantum system \((\mathcal{H}, H, \rho)\)

\[ H = H_0 + V \]

Energy conservation: \(H_{0,t} - H_0 = -V_t + V = (\neg V)_t - (\neg V)\)

as operator on \(\mathcal{H}\) yields an identity between all spectral measures but

\[ \mathbb{P}_{H_{0,t}} \neq \mathbb{P}_{-V,t} \]
Two-time measurement statistics

Conservation laws—energy balance

Classical system \((\mathcal{M}, H, \rho)\)

\[ H = H_0 + V \]

\(\rho\) is invariant for the dynamics associated to \(H_0\)

\((H_0\) interpreted as heat\)

Energy conservation:

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as function on \(\mathcal{M}\)

In particular if \(V\) is bounded by \(C\) then

\[ \sup_t |\Delta H_{0,t}| < 2C \]

and \(\text{supp}(\mathbb{P}_{\Delta H_{0,t}})\) bounded

Quantum system \((\mathcal{H}, H, \rho)\)

\[ H = H_0 + V \]

Energy conservation:

\[ H_{0,t} - H_0 = -V_t + V = (-V)_t - (-V) \]
as operator on \(\mathcal{H}\) yields an identity between all spectral measures but

\[ \mathbb{P}_{H_0,t} \neq \mathbb{P}_{-V,t} \]

A priori one can have: \(\text{supp}(\mathbb{P}_{V,t})\) bounded but

\[ \mathbb{E}_t(\phi^{2n}) = \int \phi^{2n} d\mathbb{P}_{H_0,t}(\phi) = +\infty \]
Two-time measurement statistics

Conservation laws-energy balance

Classical system $(\mathcal{M}, H, \rho)$

$H = H_0 + V$ \quad $\rho$ is invariant for the dynamics associated to $H_0$ ($H_0$ interpreted as heat)

Energy conservation:

$\Delta H_{0,t} = H_{0,t} - H_0 = -(V_t - V) = (-V)_t - (-V)$ as function on $\mathcal{M}$

In particular if $V$ is bounded by $C$ then

$\sup_t |\Delta H_{0,t}| < 2C$ and $\text{supp}(\mathbb{P}_{\Delta H_0})$ bounded

Quantum system $(\mathcal{H}, H, \rho)$ \quad $H = H_0 + V$

Energy conservation: $H_{0,t} - H_0 = -V_t + V = (-V)_t - (-V)$ as operator on $\mathcal{H}$ yields an identity between all spectral measures but

$\mathbb{P}_{H_0,t} \neq \mathbb{P}_{-V,t}$

A priori one can have: $\text{supp}(\mathbb{P}_{V,t})$ bounded but

$\mathbb{E}_t(\phi^{2n}) = \int \phi^{2n} d\mathbb{P}_{H_0,t}(\phi) = +\infty$

We want to study the behaviour of $\mathbb{P}_{H_0,t}$ tails
Two-time measurement statistics

Conservation laws-energy balance

Quantum system $(\mathcal{H}, H, \rho)$ \hspace{1cm} $H = H_0 + V$

Energy conservation:

$H_0, t - H_0 = (-V)t - (-V)$

Work definition: lack of notion of trajectory

Work defined as energy/heat difference

$W := H_0, t - H_0 = (-V)t - (-V)$

 Leads to

1. no-fluctuation relations
2. $\mu \Delta H_0 = \mu W$

Work is not an observable

Work defined as energy/heat difference between two measurements

1. fluctuation relations
2. $P_{H_0, t} \neq P_{-V, t}$

Underlying jump picture
Two-time measurement statistics

Conservation laws-energy balance

Quantum system \((\mathcal{H}, H, \rho)\) \[ H = H_0 + V \]

Energy conservation: \[ H_{0,t} - H_0 = -V_t + V = (-V)_t - (-V) \] as operator on \(\mathcal{H}\)
Two-time measurement statistics

Conservation laws-energy balance

Quantum system \( (\mathcal{H}, H, \rho) \) \( H = H_0 + V \)

Energy conservation: \( H_{0,t} - H_0 = -V_t + V = (-V)_t - (-V) \) as operator on \( \mathcal{H} \)

Work definition (?): lack of notion of trajectory

Work defined as energy/heat difference

\( H_{0,t} - H_0 = -V_t + V = (-V)_t - (-V) \) as operator on \( \mathcal{H} \)

Work definition (?): lack of notion of trajectory

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Conservation laws - energy balance

Quantum system \((\mathcal{H}, H, \rho)\) \[ H = H_0 + V \]

Energy conservation: \[ H_{0,t} - H_0 = -V_t + V = (-V)_t - (-V) \] as operator on \(\mathcal{H}\)

Work definition (?): lack of notion of trajectory

Work defined as energy/heat difference

- Work as observable
  - Defining \( W := H_{0,t} - H_0 = (-V_t) - (-V) \)
    - \([\text{Bochkov-Kuzovlev '70s-'80s}]\)
  - leads to
    1. *no-fluctuation relations*!!!!
    2. \( \mu_{\Delta H_0} = \mu_W \)
Two-time measurement statistics

Conservation laws—energy balance
Quantum system \((\mathcal{H}, H, \rho)\) \(H = H_0 + V\)

Energy conservation: \(H_{0,t} - H_0 = -V_t + V = (-V)_t - (-V)\) as operator on \(\mathcal{H}\)

Work definition (??): lack of notion of trajectory

Work defined as energy/heat difference

- Work as observable
  Defining \(W := H_{0,t} - H_0 = (-V_t) - (-V)\)
  ([Bochkov-Kuzovlev ’70s–’80s])
  leads to
  1. no-fluctuation relations!!!!
  2. \(\mu_{\Delta H_0} = \mu_W\)

- Work is not an observable
  Work defined as energy/heat difference between two measurements
  1. fluctuation relations
  2. \(\mathbb{P}_{H_0,t} \neq \mathbb{P}_{-V,t}\)

Underlying jump picture
Heat fluctuations
Perturbation $V$ bounded

**General setting**

$(\mathcal{O}, \tau^t, \omega)$ $C^*$-dynamical system

$\tau^t = \tau^t_0 + i[\ - , \ V]$ and $\omega$ is a $\tau^t_0$ invariant state

$\pi_\omega : \mathcal{O} \to \mathcal{B}(\mathcal{H}_\omega)$ a GNS representation $\omega(A) = (\Omega_\omega, A\Omega_\omega)_{\mathcal{H}_\omega}$

Liouvillian: $\tau^t_0(A) = e^{+itL} \pi_\omega(A)e^{-itL}$ and $L\Omega_\omega = 0$
Heat fluctuations

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General setting

$(\mathcal{O}, \tau^t, \omega)$ $C^*$-dynamical system

$\tau^t = \tau^t_0 + i[-, V]$ and $\omega$ is a $\tau^t_0$ invariant state

$\pi_\omega : \mathcal{O} \rightarrow \mathcal{B}(\mathcal{H}_\omega)$ a GNS representation $\omega(A) = (\Omega_\omega, A\Omega_\omega)_{\mathcal{H}_\omega}$

Liouvillean: $\tau^t_0(A) = e^{+itL}\pi_\omega(A)e^{-itL}$ and $L\Omega_\omega = 0$

Definition

We define the *heat two-time measurement statistics* for time $t$, denoted $\mathbb{P}_t$, to be the spectral measure for the operator

$$L + \pi_\omega(V) - \pi_\omega(\tau^t(V)),$$

with respect to the vector $\Omega_\omega$
Heat fluctuations
Perturbation $V$ bounded

General setting

$$(\mathcal{O}, \tau^t, \omega) \text{ $C^*$-dynamical system}$$

$$\tau^t = \tau^t_0 + i[-, V] \text{ and } \omega \text{ is a } \tau^t_0 \text{ invariant state}$$

$$\pi_\omega : \mathcal{O} \rightarrow \mathcal{B}(\mathcal{H}_\omega) \text{ a GNS representation } \omega(A) = (\Omega_\omega, A\Omega_\omega)_{\mathcal{H}_\omega}$$

Liouvillean: $\tau^t_0(A) = e^{+itL}\pi_\omega(A)e^{-itL}$ and $L\Omega_\omega = 0$

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Remark

$$\mathbb{E}_t(\phi^{2n}) = \|(L + \pi_\omega(V) - \pi_\omega(\tau^t(V)))^n\Omega_\omega\|$$
Heat fluctuations
Perturbation $V$ bounded

**Theorem (Benoist, P., Raquépas 2017)**

Let $(\mathcal{O}, \tau, \omega)$ be a $C^*$-dynamical system as above and $\mathbb{P}_t$ be the heat full statistics measure associated to a self-adjoint perturbation $V \in \mathcal{O}$. Then

$$(nD) \Rightarrow \sup_{t \in \mathbb{R}} \mathbb{E}_t[\phi^{2n+2}] < +\infty.$$ 

$$(\gamma A) \Rightarrow \sup_{t \in \mathbb{R}} \mathbb{E}_t[e^{\gamma|\phi|}] \leq +\infty.$$ 

**Corollary**

Under the conditions of the previous theorem,

$$(nD) \Rightarrow \mathbb{P}_t(|\phi| \geq tR) \leq C_n(Rt)^{-2n+2}$$

$$(\gamma A) \Rightarrow \mathbb{P}_t(|\phi| \geq tR) \leq C_{\gamma} e^{-Rt}.$$
Heat fluctuations

Regularity condition optimality

Quantum impurity in a free Fermi gas

\[ \mathcal{H} = \Gamma_a(\mathbb{C}) \otimes \Gamma_a(L^2(\mathbb{R}_+, \text{d}e)) = \mathbb{C}^2 \otimes \Gamma_a(L^2(\mathbb{R}_+, \text{d}e)) \]

\[ H_0 = d\Gamma(\varepsilon_0) \otimes 1_\mathcal{H} + 1_\mathcal{H} \otimes d\Gamma(\hat{e}) \]

\[ H = H_0 + (a^*(1) \otimes 1_\mathcal{H})(1_\mathcal{H} \otimes a(f)) + (a(1) \otimes 1_\mathcal{H})(1_\mathcal{H} \otimes a^*(f)) \]

\[ f \in L^2(\mathbb{R}_+, \text{d}e) \]

\[ \omega \text{ is a } (\tau_0, \beta) \text{ KMS state} \]
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\[ \mathcal{H} = \Gamma_a(\mathbb{C}) \otimes \Gamma_a(L^2(\mathbb{R}_+, \, d\epsilon)) = \mathbb{C}^2 \otimes \Gamma_a(L^2(\mathbb{R}_+, \, d\epsilon)) \]

\[ H_0 = d\Gamma(\varepsilon_0) \otimes 1 + 1 \otimes d\Gamma(\hat{\epsilon}) \]

\[ H = H_0 + (a^*(1) \otimes 1)(1 \otimes a(f)) + (a(1) \otimes 1)(1 \otimes a^*(f)) \]

\[ f \in L^2(\mathbb{R}_+, \, d\epsilon) \]

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Theorem (Benoist, P., Raquépas 2017)

For the above model the following are equivalent:

1. \( \sup_{t \in \mathbb{R}} E_t[\phi^{2n+2}] < \infty; \)

2. for a non-trivial time interval \([t_1, t_2]\) \( \int_{t_1}^{t_2} E_t[\phi^{2n+2}] \, dt < \infty; \)

3. \((nD)\)
Heat fluctuations

Regularity condition optimality

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For this model \((nD)\) is equivalent to \( \mathbb{R} \ni s \mapsto e^{is\hat{e}} f \in L^2(\mathbb{R}^+, de) \) is \(n\) times norm- differentiable i.e \( f \in \text{Dom}(\hat{e}^n) \)
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Theorem (Benoist, P., Raquépas 2017)

*For the above model the following are equivalent:*

1. \[ \sup_{t \in \mathbb{R}} E_t[\phi^{2n+2}] < \infty; \]
2. \[ \text{for a non-trivial time interval } [t_1, t_2] \int_{t_1}^{t_2} E_t[\phi^{2n+2}] \, dt < \infty; \]
3. \( (nD) \)

For this model \( (nD) \) is equivalent to \[ \mathbb{R} \ni s \mapsto e^{is\hat{\varepsilon}} f \in L^2(\mathbb{R}^+, de) \]
is \( n \) times norm- differentiable i.e \( f \in \text{Dom}(\hat{\varepsilon}^n) \)

Remark: decay of \( f \) controls how high energy frequencies contribute to the interaction (ultraviolet regularity)
Heat exchange between two reservoirs

\[ H_0 = H_1 + H_2, \ V \text{ small} \]

Classical case:

\[ P_{1/t} H_0, t \approx 0 \text{ and } P_{1/t} H_1, t \approx P_{-1/t} H_2, t \]

Statements about control of fluctuation (large deviation principle) are trivially satisfied.

Quantum setting:

\[ P_{1/t} H_0, t \approx 0 \text{ NOT TRUE ANYMORE} \]
\[ P_{1/t} H_1, t \approx P_{1/t} H_2, t \text{ has to be justified} \]

Theorem (Benoist, Pautrat, P. "19)

Under stronger analiticity properties (UV conditions), usual statement of control of fluctuation are still true (law of large number, central limit theorem, large deviation principle).
Conclusions:

- **Classical statistical mechanics models:** energy fluctuations are controlled by the **interaction intensity**. Particularly, **no large fluctuations exist when the interaction is bounded.**

- **Quantum statistical mechanics models:** in the two time measurement picture, energy fluctuations are controlled by a notion of regularity. Particularly, **large fluctuations may exists even if the interaction is bounded.**

  In concrete models, regularity notion translate in **contribution of high energy frequencies to the interaction (UV regularization).**

  In other words, in the quantum case the primary contribution to the fluctuations is given by **energy transitions induced by the interaction** rather then the interaction itself.
Thank you for your attention!