Heat fluctuations in the two-time measurement framework and ultraviolet regularity

joint work with T.Benoist, (Y. Pautrat) and R. Raquépas

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Context: quantum statistical mechanics

Fluctuation relations

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Classical case: [Evans-Cohen-Morris '93] numerical experiences

[Evans-Searls '94] [Gallavotti Cohen '94] theoretical explanation

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Quantum case: ??

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At the level of averages and variances, there is no difference!

Two-time measurment statistics

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Full (Counting) Statistics [Lesovik, Levitov '93][Levitov, Lee,Lesovik '96]

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Confined systems: described by $(\mathcal{H}, H, \rho)$ $dim\mathcal{H} < \infty$
Given an observable A: $A = \sum_j a_j P_{a_j}$ where $a_j \in \sigma(A)$ P_{a_j} associated spectral projections

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Two-time measurement distribution of A:
$\mathbb{P}_{A,t}(\phi)$= probability of measuring a change in A equal to ϕ.

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Conservation laws

[1] Given a classical observable C and an intial state ρ, we call C-statistics the probability measure $\mathbb{P}_C$ such that $\int f(s)\mathrm{d}\mathbb{P}_C(s) = \int f(C)\mathrm{d}\rho$ for all $f \in \mathcal{B}(\mathbb{R})$

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Quantum system $A_t - A = -(B_t - B)$ as operators yields identity between all spectral measures

$$\mu_{\triangle A_t} = \mu_{-\triangle B_t} \quad \text{but} \quad \mathbb{P}_{A,t} \neq \mathbb{P}_{-B,t}$$

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Classical system $(\mathcal{M}, H, \rho)$

$H = H_0 + V$ $\quad$ ρ is invariant for the dynamics associated to H_0

(H_0 interpreted as heat)

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We want to study the behaviour of $\mathbb{P}_{H_0,t}$ tails

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Work defined as energy/heat difference

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  Defining $W := H_{0,t} - H_0 = (-V_t) - (-V)$
  ([Bochkov-Kuzovlev '70s-'80s])
  leads to
  1. *no-fluctuation relations*!!!!
  2. $\mu_{\Delta H_0} = \mu_W$

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- Work is not an observable
  Work defined as energy/heat difference between two measurments
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  Underlying jump picture

Heat fluctuations

Perturbation V bounded

General setting

$(\mathcal{O}, \tau^t, \omega)$ C^*-dynamical system

$\tau^t = \tau_0^t + \mathrm{i}[-, V]$ and ω is a τ_0^t invariant state

$\pi_\omega : \mathcal{O} \to \mathcal{B}(\mathcal{H}_\omega)$ a GNS representation $\omega(A) = (\Omega_\omega, A\Omega_\omega)_{\mathcal{H}_\omega}$

Liouvillean: $\tau_0^t(A) = \mathrm{e}^{+\mathrm{i}tL}\pi_\omega(A)\mathrm{e}^{-\mathrm{i}tL}$ and $L\Omega_\omega = 0$

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Remark

$\mathbb{E}_t(\phi^{2n}) = \|(L + \pi_\omega(V) - \pi_\omega(\tau^t(V)))^n \Omega_\omega\|$

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Theorem (Benoist, P., Raquépas 2017)

Let $(\mathcal{O}, \tau, \omega)$ be a C^*-dynamical system as above and $\mathbb{P}_t$ be the heat full statistics measure associated to a self-adjoint perturbation $V \in \mathcal{O}$. Then

$$(nD) \Rightarrow \sup_{t \in \mathbb{R}} \mathbb{E}_t[\phi^{2n+2}] < +\infty.$$

$$(\gamma A) \Rightarrow \sup_{t \in \mathbb{R}} \mathbb{E}_t[\mathrm{e}^{\gamma|\phi|}] \leq +\infty.$$

Corollary

Under the conditions of the previous theorem,

$$(nD) \Rightarrow \mathbb{P}_t(|\phi| \geq tR) \leq C_n (Rt)^{-2n+2}$$

$$(\gamma A) \Rightarrow \mathbb{P}_t(|\phi| \geq tR) \leq C_\gamma e^{-Rt}.$$

Heat fluctuations

Regularity condition optimality

Quantum impurity in a free Fermi gas

$\mathcal{H} = \Gamma_a(\mathbb{C}) \otimes \Gamma_a(L^2(\mathbb{R}_+, \mathrm{d}e)) = \mathbb{C}^2 \otimes \Gamma_a(L^2(\mathbb{R}_+, \mathrm{d}e))$

$H_0 = \mathrm{d}\Gamma(\varepsilon_0) \otimes \mathbb{1} + \mathbb{1} \otimes \mathrm{d}\Gamma(\hat{e})$

$H = H_0 + (a^*(1) \otimes \mathbb{1})(\mathbb{1} \otimes a(f)) + (a(1) \otimes \mathbb{1})(\mathbb{1} \otimes a^*(f))$

$f \in L^2(\mathbb{R}_+, \mathrm{d}e)$

ω is a (τ_0, β) KMS state

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Theorem (Benoist, P., Raquépas 2017)

For the above model the following are equivalent:

1. $\sup_{t\in\mathbb{R}} \mathbb{E}_t[\phi^{2n+2}] < \infty$;
2. *for a non-trivial time interval* $[t_1, t_2]$ $\int_{t_1}^{t_2} \mathbb{E}_t[\phi^{2n+2}]\mathrm{d}t < \infty$;
3. (nD)

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$H_0 = \mathrm{d}\Gamma(\varepsilon_0) \otimes \mathbb{1} + \mathbb{1} \otimes \mathrm{d}\Gamma(\hat{e})$
$H = H_0 + (a^*(1) \otimes \mathbb{1})(\mathbb{1} \otimes a(f)) + (a(1) \otimes \mathbb{1})(\mathbb{1} \otimes a^*(f))$
$f \in L^2(\mathbb{R}_+, \mathrm{d}e)$
ω is a (τ_0, β) KMS state

Theorem (Benoist, P., Raquépas 2017)

For the above model the following are equivalent:

1. $\sup_{t\in\mathbb{R}} \mathbb{E}_t[\phi^{2n+2}] < \infty$;
2. *for a non-trivial time interval* $[t_1, t_2]$ $\int_{t_1}^{t_2} \mathbb{E}_t[\phi^{2n+2}]\mathrm{d}t < \infty$;
3. (nD)

For this model (nD) is equivalent to $\mathbb{R} \ni s \mapsto \mathrm{e}^{\mathrm{i}s\hat{e}} f \in L^2(\mathbb{R}_+, \mathrm{d}e)$ is n times norm- differentiable i.e $f \in \mathrm{Dom}(\hat{e}^n)$

Heat fluctuations

Regularity condition optimality

Quantum impurity in a free Fermi gas

$\mathcal{H} = \Gamma_a(\mathbb{C}) \otimes \Gamma_a(L^2(\mathbb{R}_+, \mathrm{d}e)) = \mathbb{C}^2 \otimes \Gamma_a(L^2(\mathbb{R}_+, \mathrm{d}e))$
$H_0 = \mathrm{d}\Gamma(\varepsilon_0) \otimes \mathbb{1} + \mathbb{1} \otimes \mathrm{d}\Gamma(\hat{e})$
$H = H_0 + (a^*(1) \otimes \mathbb{1})(\mathbb{1} \otimes a(f)) + (a(1) \otimes \mathbb{1})(\mathbb{1} \otimes a^*(f))$
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Remark: decay of f controls how high energy frequencies contribute to the interaction (ultraviolet regularity)

Heat exchange between two reservoirs

$H_0 = H_1 + H_2$, V small

Classical case:

$\mathbb{P}_{\frac{1}{t}H_0,t} \approx 0$ and $\mathbb{P}_{\frac{1}{t}H_1,t} \approx \mathbb{P}_{-\frac{1}{t}H_2,t}$

statements about control of fluctuation (large deviation principle) are trivially satisfied

Quantum setting:

$\mathbb{P}_{\frac{1}{t}H_0,t} \approx 0$ NOT TRUE ANYMORE

$\mathbb{P}_{\frac{1}{t}H_1,t} \approx \mathbb{P}_{\frac{1}{t}H_2,t}$ has to be justified

Theorem (Benoist, Pautrat, P. "19)

Under stronger analiticity properties (UV conditions), usual statement of control of fluctuation are still true (law of large number, central limit theorem, large deviation principle).

Conclusions:

- Classical statistical mechanics models: energy fluctuations are controlled by the **interaction intensity**. Particularly, **no large fluctuations exist when the interaction is bounded**.
- Quantum statistical mechanics models: in the two time measurement picture, energy fluctuations are controlled by a notion of regularity. Particularly, **large fluctuations may exists even if the interaction is bounded**.
  In concrete models, regularity notion translate in **contribtuon of high energy frequencies to the interaction** (UV regularization).
  In other words, in the quantum case the primary contribution to the fluctuations is given by **energy transitions induced by the interaction** rather then the interaction itself.

Thank you for your attention!