Universal edge transport in interacting Hall systems

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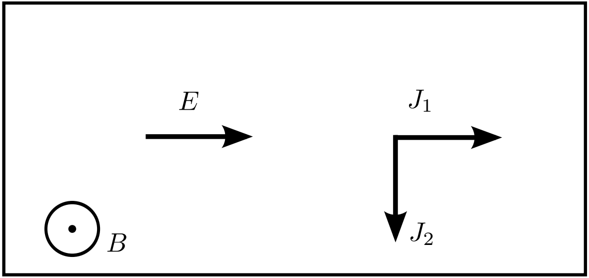
Joint work with G. Antinucci (UZH) and V. Mastropietro (Milan)

Hall effect

- Hall effect (Edwin Hall 1879):



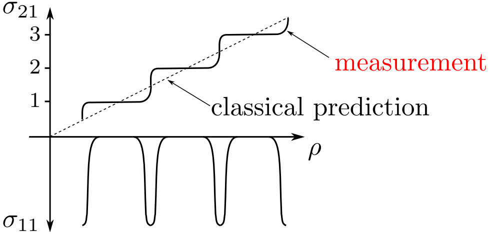
- Linear response (weak E):

$$J_1 = \sigma_{11} E \, , \qquad J_2 = \sigma_{21} E \, .$$

σ_{11} = longitudinal conductivity, $\sigma_{21} = -\sigma_{12}$ = Hall conductivity.

Integer quantum Hall effect

- von Klitzing '80. Experiment on GaAs-heterostructures (insulators).



($\rho =$ density of charge carriers.) IQHE: $\sigma_{21} = \frac{e^2}{h} \cdot n$, $n \in \mathbb{Z}$.

- Theory for noninteracting systems: Laughlin '81, Thouless *et al.* '82 ...
  Rigorous results: Avron-Seiler-Simon '83, Bellissard *et al.* '94, Aizenman-Graf '98 ...

Bulk-edge correspondence

- Halperin '82. Hall phases come with robust edge currents.

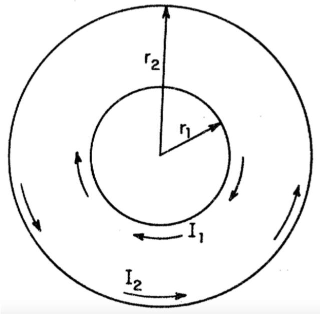


Figure: Magnetic field points out of the screen.

- Edge currents are necessary to preserve gauge invariance.
  Essential feature of the gauge theory of states of matter [Fröhlich '91]

Bulk-edge correspondence: rigorous results

- Halperin '82. Hall phases come with robust edge currents.
- Hatsugai '93; Schulz-Baldes *et al* '00, Graf *et al.* '02: bulk-edge duality.

$$\sigma_{12} = \frac{e^2}{h} \sum_e \omega_e \qquad \omega_e = (\text{chirality of the edge state}) \in \{-1, +1\}$$

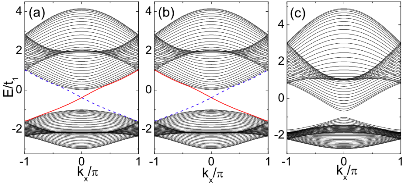


Figure: $(a) : \sigma_{12} = \frac{e^2}{h}, \qquad (b) : \sigma_{12} = -\frac{e^2}{h}, \qquad (c) : \sigma_{12} = 0.$

- Graf-P. '13: extension to quantum spin Hall systems.

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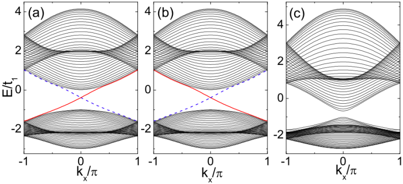

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- Graf-P. '13: extension to quantum spin Hall systems.
- Many-body interactions?

Interacting systems

Lattice fermions

- Interacting electron gas on $\Lambda_L = [0,L]^2 \subset \mathbb{Z}^2$. Fock space Hamiltonian:

$$\mathcal{H} = \sum_{\vec{x},\vec{y}} \sum_{\rho,\rho'} a^+_{\vec{x},\rho} H_{\rho\rho'}(\vec{x},\vec{y}) a^-_{\vec{x},\rho'} + \lambda \sum_{\vec{x},\vec{y}} \sum_{\rho,\rho'} n_{\vec{x},\rho} v_{\rho\rho'}(\vec{x},\vec{y}) n_{\vec{y},\rho'} - \mu\mathcal{N}$$

  H, v finite-ranged, $\rho \in \{1,\ldots,M\}$ = internal degree of freedom.

- H is equipped with cylindric boundary conditions:

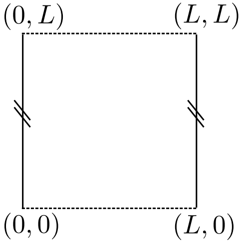

Figure: Dotted lines: Dirichlet boundary conditions.

- Translation invariance in x_1 direction: $H_{\rho\rho'}(\vec{x},\vec{y}) \equiv H_{\rho\rho'}(x_1 - y_1; x_2, y_2)$.

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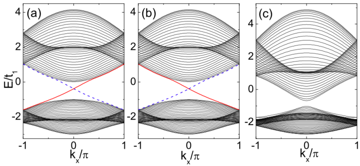
  H, v finite-ranged, $\rho \in \{1, \ldots, M\}$ = internal degree of freedom.

- Assumption. For periodic b.c., $\sigma(H^{(\mathrm{per})})$ is gapped.
  Instead, edge states might appear in $\sigma(H)$.

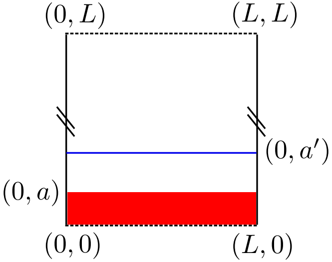


- $\varepsilon(k_1)$ = eigenvalue branch of $\hat{H}(k_1)$. The corresponding edge state is:

$$\hat{\varphi}_{\vec{x}}(k_1) = e^{ik_1 x_1} \xi_{x_2}(k_1) \;, \qquad \text{with } \xi_{x_2}(k_1) \sim e^{-cx_2} \;.$$

Edge transport coefficients

- Let $\mu \in \sigma(H^{(\mathrm{per})})$.
- Edge transport. Perturb at distance $\leq a$ from $x_2 = 0$. Linear response?



- Interesting physical observables: charge density and current density,

$$n_{\vec{x}} = \sum_{\rho} a^{+}_{\vec{x},\rho} a^{-}_{\vec{x},\rho} \; , \qquad \vec{j}_{\vec{x}} = \sum_{i=1,2} \sum_{\rho,\rho'} \vec{e}_i [i a^{+}_{\vec{x}+\vec{e}_i,\rho} H_{\rho\rho'}(\vec{x}+\vec{e}_i, \vec{x}) a^{-}_{\vec{x},\rho'} + \mathrm{h.c.}] \; .$$

Their support will be $x_2 \leq a'$, with $L \gg a' \gg a \gg 1$.

Edge transport coefficients

- Let $\mu \in \sigma(H^{(\mathrm{per})})$.
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- Edge charge susceptibility:

$$\kappa^{a,a'}(\eta, p_1) \quad = \quad i \int_{-\infty}^{0} dt\, e^{t\eta} \big\langle [\hat{n}^{\leq a}_{p_1}(t)\,, \hat{n}^{\leq a'}_{-p_1}] \big\rangle_\infty$$

$\hat{n}^{\leq a}_{p_1} = \sum_{x_2 \leq a} \hat{n}_{p_1, x_2}, \quad \langle \cdot \rangle_\infty = \lim_{\beta, L \to \infty} L^{-1} \mathrm{Tr} \cdot e^{-\beta(\mathcal{H} - \mu \mathcal{N})} / \mathcal{Z}_{\beta, L}.$

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- Charge conductance and Drude weight:

$$G^{a,a'}(\eta, p_1) \quad = \quad i \int_{-\infty}^{0} dt\, e^{t\eta} \left\langle [\hat{n}^{\leq a}_{p_1}(t)\,, \hat{j}^{\leq a'}_{1,-p_1}] \right\rangle_{\infty}$$

$$D^{a,a'}(\eta, p_1) \quad = \quad -i \Big[\int_{-\infty}^{0} dt\, e^{t\eta} \left\langle [\hat{j}^{\leq a}_{1,p_1}(t)\,, \hat{j}^{\leq a'}_{1,-p_1}] \right\rangle_{\infty} + \Delta^a \Big]$$

with $\Delta^a = \langle [X^{\leq a}_1, \hat{j}^{\leq a}_{1,0}] \rangle_{\infty}$. Spin transport: $n \to n_{\uparrow} - n_{\downarrow}, \quad j \to j_{\uparrow} - j_{\downarrow}$.

Bulk transport coefficients

- The response to bulk perturbations is expected to be edge-independent.
- Kubo formula: bulk conductivity matrix.

$$\sigma_{ij} = \lim_{\eta\to 0^+} \frac{i}{\eta}\Big[\int_{-\infty}^{0} dt\, e^{\eta t}\,\big\langle [j_i(t)\,, j_j]\big\rangle_\infty^{(\mathrm{per})} + \big\langle [X_i\,, j_j]\big\rangle_\infty^{(\mathrm{per})}\Big]$$

$$\vec{j} = \textstyle\sum_{\vec{x}} \vec{j}_{\vec{x}}, \qquad \langle\,\cdot\,\rangle_\infty^{(\mathrm{per})} = \lim_{\beta,L\to\infty} L^{-2}\mathrm{Tr}\,\cdot e^{-\beta(\mathcal{H}^{(\mathrm{per})}-\mu\mathcal{N})}/\mathcal{Z}_{\beta,L}^{(\mathrm{per})}$$

- Bachmann-de Roeck-Fraas '17, Monaco-Teufel '17: derivation of Kubo formula for gapped many-body lattice models.
- Hastings-Michalakis '14, Giuliani-Mastropietro-P. '15: quantization of σ_{12} for $\lambda \neq 0$. [HM]: gapped $\mathcal{H}$. [GMP]: $\lambda \ll \mathrm{gap}(H)$.
- Giuliani-Jauslin-Mastropietro-P. '16: Hall transitions in the Haldane-Hubbard model: $\lambda \gg \mathrm{gap}(H)$.

Noninteracting edge transport

- Let $\lambda = 0$. Define $\underline{p} = (\eta, p_1)$. Edge transport coefficients:

$$\kappa^{a,a'}(\underline{p}) = \sum_e \frac{\omega_e}{2\pi} \frac{p_1}{-i\eta + v_e p_1} + R_\kappa^{a,a'}(\underline{p}) \qquad \text{(susceptivity)}$$

$$G^{a,a'}(\underline{p}) = \sum_e \frac{\omega_e}{2\pi} \frac{-i\eta}{-i\eta + v_e p_1} + R_G^{a,a'}(\underline{p}) \qquad \text{(conductance)}$$

$$D^{a,a'}(\underline{p}) = \sum_e \frac{|v_e|}{2\pi} \frac{-i\eta}{-i\eta + v_e p_1} + R_D^{a,a'}(\underline{p}) \qquad \text{(Drude weight)}$$

$v_e =$ velocity of edge state, $\omega_e = \mathrm{sgn}(v_e)$, $\lim_{a,a' \to \infty} \lim_{\underline{p} \to 0} R_\sharp^{a,a'}(\underline{p}) = 0$.

- Bulk-edge correspondence:

$$G = \lim_{a,a' \to \infty} \lim_{\eta \to 0^+} \lim_{p_1 \to 0} G^{a,a'}(\eta, p_1) = \sum_e \frac{\omega_e}{2\pi} = \sigma_{12}$$

Interacting edge transport: Bosonization

- Edge transport coefficients $\sim$ correlations of $\partial_{x_0}\phi^e$, $\partial_{x_1}\phi^e$, with ϕ^e = bosonic free field with covariance [Wen, '90]:

$$\langle \hat{\phi}^{e+}_{\underline{p}} \hat{\phi}^{e'-}_{-\underline{p}} \rangle = \delta_{ee'} \frac{\omega_e}{2\pi} \frac{1}{p_1(-i\eta + v_e p_1)} \ .$$

- Bosonization. Mapping of interacting, relativistic fermions in free bosons with interaction-dependent parameters: "$n_e \to \partial_{x_1}\phi^e$".

- The mapping in free bosons breaks down for nonrelativistic models. Nonlinearities produce quartic interactions among bosons.

- Exact computation of the transport coefficients for $\lambda \neq 0$?

- From now on: one edge state per edge (spin degenerate).

Reference model: Chiral Luttinger liquid

- Chiral Luttinger liquid. Massless $1+1$-dim. Grassmann field:

$$S_0^{(\mathrm{ref})}(\psi) = Z \int_{\mathbb{R}^2} \frac{d\underline{k}}{(2\pi)^2}\, \psi^+_{\underline{k},\sigma}(-ik_0 + vk_1)\psi^-_{\underline{k},\sigma}\ .$$

Noninteracting density-density correlation function:

$$\langle \hat{n}_{\underline{p}}\,;\hat{n}_{-\underline{p}}\rangle^{(\mathrm{ref})} = -\frac{1}{2\pi|v|Z^2}\frac{-ip_0 - vp_1}{-ip_0 + vp_1}\ .$$

- Expect. Lattice edge states effectively described by interacting χLL:

$$S^{(\mathrm{ref})}(\psi) = Z \int_{\mathbb{R}^2} \frac{d\underline{k}}{(2\pi)^2}\, \psi^+_{\underline{k},\sigma}(-ik_0 + vk_1)\psi^-_{\underline{k},\sigma} + gZ^2 \int_{\mathbb{R}^2} \frac{d\underline{p}}{(2\pi)^2}\, \hat{w}(\underline{p})\hat{n}_{\underline{p}}\hat{n}_{-\underline{p}}$$

for suitable bare parameters Z, v, g (and suitable regularization scheme).

Theorem 1/2: Charge transport coefficients

Theorem (Antinucci-Mastropietro-P. '17)

There exists λ_0 s.t. for $|\lambda| < \lambda_0$:

$$\begin{aligned}
\kappa^{a,a'}(\eta, p_1) &= \frac{\omega}{\pi}\frac{p_1}{-i\eta + v_c(\lambda)p_1} + R_\kappa^{a,a'}(\underline{p}) \\
G^{a,a'}(\eta, p_1) &= \frac{\omega}{\pi}\frac{-i\eta}{-i\eta + v_c(\lambda)p_1} + R_G^{a,a'}(\underline{p}) \\
D^{a,a'}(\eta, p_1) &= \frac{|v_c(\lambda)|}{\pi}\frac{-i\eta}{-i\eta + v_c(\lambda)p_1} + R_D^{a,a'}(\underline{p})
\end{aligned}$$

$v_c(\lambda) =$ *renormalized charge velocity,* $\lim_{a,a'\to\infty}\lim_{\underline{p}\to 0} R_\sharp^{a,a'}(\underline{p}) = 0$

Rmks.

- $v_c(\lambda) =$ analytic function of λ, given by an explicit, convergent series.
- Results valid for the full lattice model.
- Similar expressions for spin coefficients, with $v_c(\lambda) \to v_s(\lambda) \neq v_c(\lambda)$.

Theorem 2/2: Spin-charge separation

Theorem (Antinucci-Mastropietro-P. '17)

Let $\mathbf{x} = (x_0, \vec{x})$ *with* $x_0 =$ *imaginary time. For* $|\lambda| < \lambda_0$ *and* $\mathbf{x} \neq \mathbf{y}$*:*

$$\langle \mathbf{T} a^-_{\mathbf{x},\rho} a^+_{\mathbf{y},\rho'} \rangle_\infty = \frac{Z^{-1} e^{ik_F(x_1 - y_1)} \xi_{x_2}(\rho) \overline{\xi_{y_2}}(\rho')}{\sqrt{[v_s(x_0 - y_0) + i\omega(x_1 - y_1)][v_c(x_0 - y_0) + i\omega(x_1 - y_1)]}} + R_{\rho\rho'}(\mathbf{x}, \mathbf{y})$$

where: $\xi \equiv \xi^e(k_F)$*;* $\quad Z \equiv Z(\lambda) = 1 + O(\lambda)$*;* $\quad k_F \equiv k_F(\lambda) = k_F^e + O(\lambda)$

$$|R_{\rho\rho'}(\mathbf{x}, \mathbf{y})| \leq \frac{C e^{-|x_2 - y_2|}}{\|(x_0 - y_0)^2 + (x_1 - y_1)^2\|^{\frac{1}{2} + \theta}}\ , \qquad \theta > 0\ .$$

Rmks.

- $v_s =$ spin velocity, $v_c =$ charge velocity, $v_c(\lambda) - v_s(\lambda) = \frac{\lambda}{\pi} + O(\lambda^2)$
- Similar result for χLL: Falco-Mastropietro '08.

Remarks

- The result implies that:

$$G = \lim_{a,a'\to\infty} \lim_{\eta\to 0^+} \lim_{p_1\to 0^+} G^{a,a'}(\eta, p_1) \quad = \quad \frac{\omega}{\pi} \quad \underbrace{=}_{\text{bulk-edge corresp.}} \quad \sigma_{12}(\lambda = 0)$$

The interacting bulk-edge duality follows from $\sigma_{12}(0) = \sigma_{12}(\lambda)$ [GMP].

- Similarly, let:

$$\kappa = \lim_{a,a'\to\infty} \lim_{p_1,\eta\to 0^+} \kappa^{a,a'}(\eta, p_1)\;, \qquad D = \lim_{a,a'\to\infty} \lim_{\eta,p_1\to 0^+} D^{aa}(\eta, p_1).$$

Then: $\frac{D}{\kappa} = v_c^2$. "Haldane relation", valid for Luttinger liquids.

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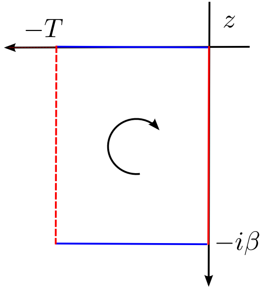
- Benfatto-Falco-Mastropietro '09–'12: Haldane relations for general, nonsolvable lattice $1d$ systems.
- Proof based on RG methods and Ward identities.

Sketch of the proof

Part 1/3: Wick rotation

- Analytic continuation to imaginary times. We have:

$$\int_{-\infty}^{0} dt\, e^{t\eta}\, \langle [\hat{n}_{p_1}^{\leq a}(t)\,,\hat{j}_{1,-p_1}^{\leq a'}]\rangle_\infty = i\int_0^\infty dt\, e^{-i\eta t} \langle \hat{n}_{p_1}^{\leq a}(-it)\,;\hat{j}_{1,-p_1}^{\leq a'}\rangle_\infty$$



- Errors (dotted red) estimated via bounds on Euclidean correlations:

$$|\langle A(T-it)B\rangle_{\beta,L}| \leq \langle A(-it)A(-it)^*\rangle_{\beta,L}^{1/2}\langle B^*B\rangle_{\beta,L}^{1/2}$$

Part 2/3: Computation of Euclidean correlations

- Model can be studied via RG. Problem: ψ^4 interactions are marginal.
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- Comparison between reference and lattice model:

$$\int_0^\infty dt\, e^{-ip_0 t} \langle \hat{n}^{\leq a}_{p_1}(-it)\, ; \hat{j}^{\leq a'}_{1,-p_1} \rangle_\infty =$$

$$\sum_{\substack{x_2 \leq a \\ y_2 \leq a'}} Z_0(x_2) Z_1(y_2) \int_0^\infty dt\, e^{-ip_0 t} \langle \hat{n}_{p_1}(-it)\, ; \hat{n}_{-p_1} \rangle^{(\mathrm{ref})} + A^{a,a'} + R^{a,a'}(\underline{p})$$

$|Z_i(x_2)| \leq C e^{-c x_2}$, $\qquad |A^{a,a'}| \leq C$, $\qquad |R^{a,a'}(\underline{p})| \leq C a |\underline{p}|^\theta$, $\qquad$ and

$$\int_0^\infty dt\, e^{-ip_0 t} \langle \hat{n}_{p_1}(-it)\, ; \hat{n}_{-p_1} \rangle^{(\mathrm{ref})} = -\frac{1}{2\pi |v_s| Z^2} \frac{1}{1+\tau} \frac{-ip_0 - v_s p_1}{-ip_0 + v_c p_1}$$

with: $\qquad v_s = v, \qquad \tau = \frac{g}{2\pi v}, \qquad \frac{v_c}{v_s} = \frac{1-\tau}{1+\tau}.$

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with: $\quad v_s = v, \qquad \tau = \frac{g}{2\pi v}, \qquad \frac{v_c}{v_s} = \frac{1-\tau}{1+\tau}.$

- We are left with computing the renormalized parameters.

Part 3/3: Ward identities

- All unknowns parameters can be computed thanks to Ward identities.
  Setting $\mathbf{x} = (t, \vec{x}) \equiv (x_0, \vec{x})$:

$$i\partial_{x_0}\langle \mathbf{T} n_{\mathbf{x}}\,; n_{\mathbf{y}}\rangle_\infty + \vec{\nabla}_{\vec{x}} \cdot \langle \mathbf{T} \vec{j}_{\mathbf{x}}\,; n_{\mathbf{y}}\rangle_\infty = 0 \Rightarrow A^\infty = -\frac{Z_0 Z_1}{2\pi v_c Z^2}\frac{1}{1+\tau}$$

with $Z_i = \sum_{x_2=0}^{\infty} Z_i(x_2)$.

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with $Z_i = \sum_{x_2=0}^{\infty} Z_i(x_2)$. Also, let $\underline{x} = (x_0, x_1)$. Vertex WIs:

$$\begin{aligned} d_\mu \langle \mathbf{T} j_{\mu,\mathbf{z}}\,; a^-_{\mathbf{y},\rho'} a^+_{\mathbf{x},\rho}\rangle_\infty &= i\big[\delta_{\mathbf{x},\mathbf{z}}\langle \mathbf{T} a^-_{\mathbf{y},\rho'} a^+_{\mathbf{x},\rho}\rangle_\infty - \delta_{\mathbf{y},\mathbf{z}}\langle \mathbf{T} a^-_{\mathbf{y},\rho'} a^+_{\mathbf{x},\rho}\rangle_\infty\big] \\ (i\partial_0 + \partial_1)\langle \mathbf{T} n_{\underline{z}}\,; \psi^-_{\underline{y},\sigma}\psi^+_{\underline{x},\sigma}\rangle^{(\mathrm{ref})} &= \frac{i}{Z(1+\tau)}\big[\delta_{\underline{x},\underline{z}}\langle \mathbf{T}\psi^-_{\underline{y},\sigma}\psi^+_{\underline{x},\sigma}\rangle^{(\mathrm{ref})} - \delta_{\underline{y},\underline{z}}\langle \mathbf{T}\psi^-_{\underline{y},\sigma}\psi^+_{\underline{x},\sigma}\rangle^{(\mathrm{ref})}\big] \end{aligned}$$

Implication: $Z_0 = Z(1+\tau), \quad Z_1 = Z v_c(1-\tau).$

- These relations allow to prove the universality of edge conductance G.

Conclusions

- From a rigorous viewpoint, a lot is known for noninteracting topological insulators, much less in the presence of many-body interactions.
- Today: interacting Hall systems with single-mode edge currents.
  - (a) Edge transport coefficients, bulk-edge duality, Haldane relations.
  - (b) Two-point function: Spin-charge separation.
- Open problems:
  - (a) Multi-edge channels topological insulators?
  - (b) Weak disorder?
  - (c) FQHE...?

Thank you!