Full Statistics of Landauer's Principle in Adiabatic Repeated Interaction Systems*

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Landauer's Principle

Landauer '61

- Small system $\mathcal{S}$ interacting with environment $\mathcal{E}$ of hamiltonian $h_{\mathcal{E}}$ at temperature β^{-1}

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Initial state ρ^{i} and Gibbs state $\xi^{\mathrm{i}} = \exp[-\beta h_{\mathcal{E}}]/Z$

Evolution of $\mathcal{S} + \mathcal{E}$: unitary op. U on $\mathcal{H}_{\mathcal{S}} \otimes \mathcal{H}_{\mathcal{E}}$

Final states $\rho^{\mathrm{f}} = \mathrm{Tr}_{\mathcal{E}}(U\rho^{\mathrm{i}} \otimes \xi^{\mathrm{i}} U^*)$, $\xi^{\mathrm{f}} = \mathrm{Tr}_{\mathcal{S}}(U\rho^{\mathrm{i}} \otimes \xi^{\mathrm{i}} U^*)$

Landauer's bound

- Decrease of entropy: $\Delta S_{\mathcal{S}} := S(\rho^{\mathrm{i}}) - S(\rho^{\mathrm{f}})$
- Increase of energy: $\Delta Q_{\mathcal{E}} := \mathrm{Tr}(h_{\mathcal{E}}\xi^{\mathrm{f}}) - \mathrm{Tr}(h_{\mathcal{E}}\xi^{\mathrm{i}})$

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- Computation: $$\boxed{\Delta S_{\mathcal{S}} + \sigma = \beta \Delta Q_{\mathcal{E}}}$$

with entropy production $\sigma = S(U\rho^{\mathrm{i}} \otimes \xi^{\mathrm{i}}\, U^* | \rho^{\mathrm{f}} \otimes \xi^{\mathrm{i}}) \geq 0$

where $S(\eta|\nu) := \mathrm{Tr}\left(\eta(\log\eta - \log\nu)\right) \geq 0$ relative entropy

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Landauer's bound: $$\boxed{\Delta Q_{\mathcal{E}} \geq \beta^{-1} \Delta S_{\mathcal{S}}}$$

Adiabatic saturation of the bound

- Infinite dimensional $\mathcal{H}_{\mathcal{E}}$ Jaksic & Pillet '14
- Time dependent Hamiltonian $h_{\mathcal{E}} + h_{\mathcal{S}} + v(t/T)$
  adiabatic scaling $T \to \infty$
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Then:

$$\lim_{T\to\infty} \sigma_T = 0$$

Full statistics in: Benoist, Fraas, Jaksic & Pillet '16

Repeated Interaction Systems

Kümmerer, Maassen '00, Attal, Pautrat '05, Bruneau, J., Merkli '06-'14

- Structured environment $\mathcal{E}$:
  Infinite chain of independent quantum "probes"

$$\mathcal{E} = \mathcal{E}_1 + \mathcal{E}_2 + \mathcal{E}_3 + \cdots \mathcal{E}_k + \mathcal{E}_{k+1} \cdots$$

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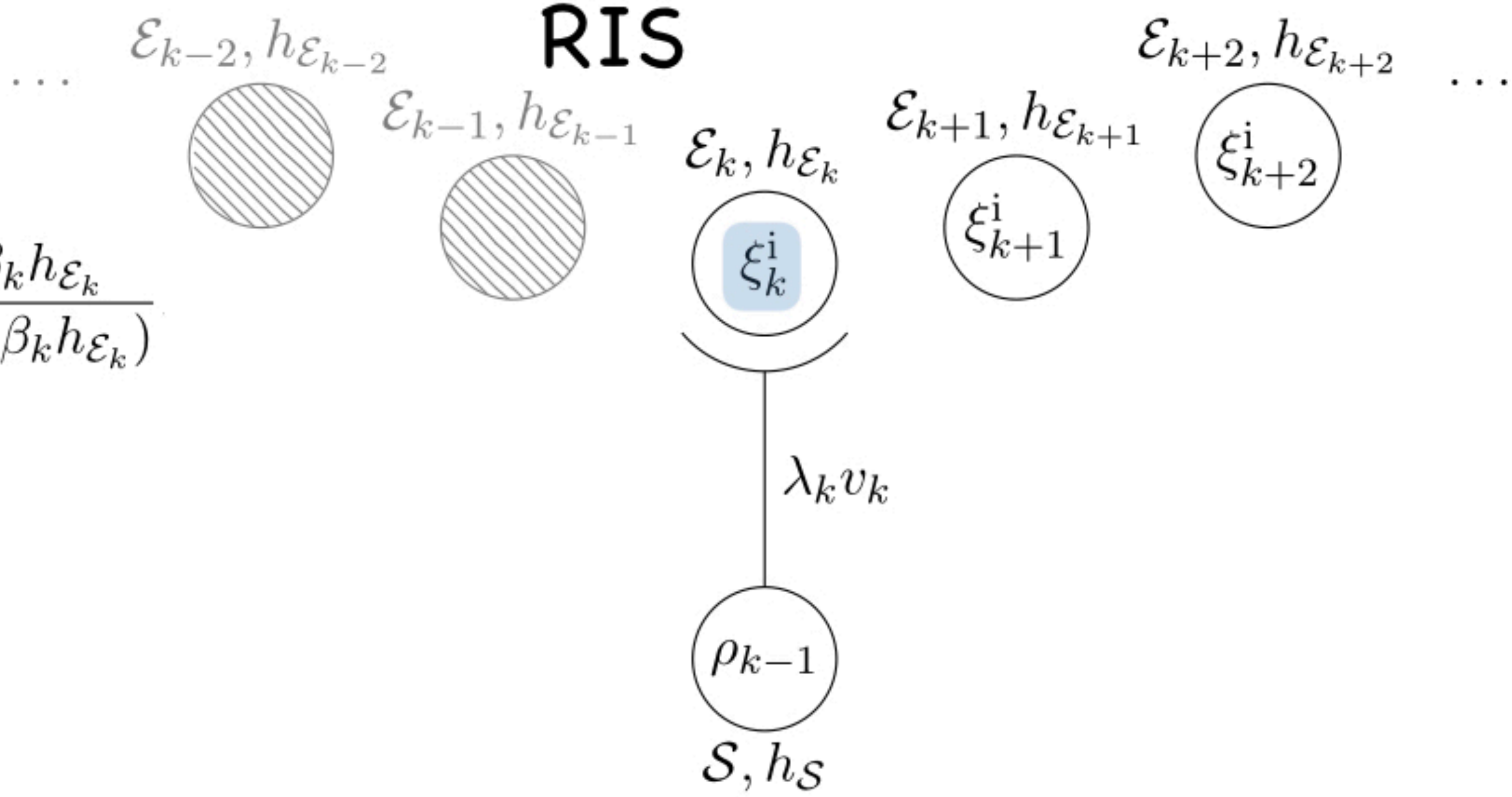
- Example: "One atom MASER"

e.g. Meschede et al '85
Haroche et al '01

RIS

Probe $\mathcal{E}_k$

$$\xi_k^{\mathrm{i}} := \frac{\exp -\beta_k h_{\mathcal{E}_k}}{\mathrm{Tr}(\exp -\beta_k h_{\mathcal{E}_k})}$$



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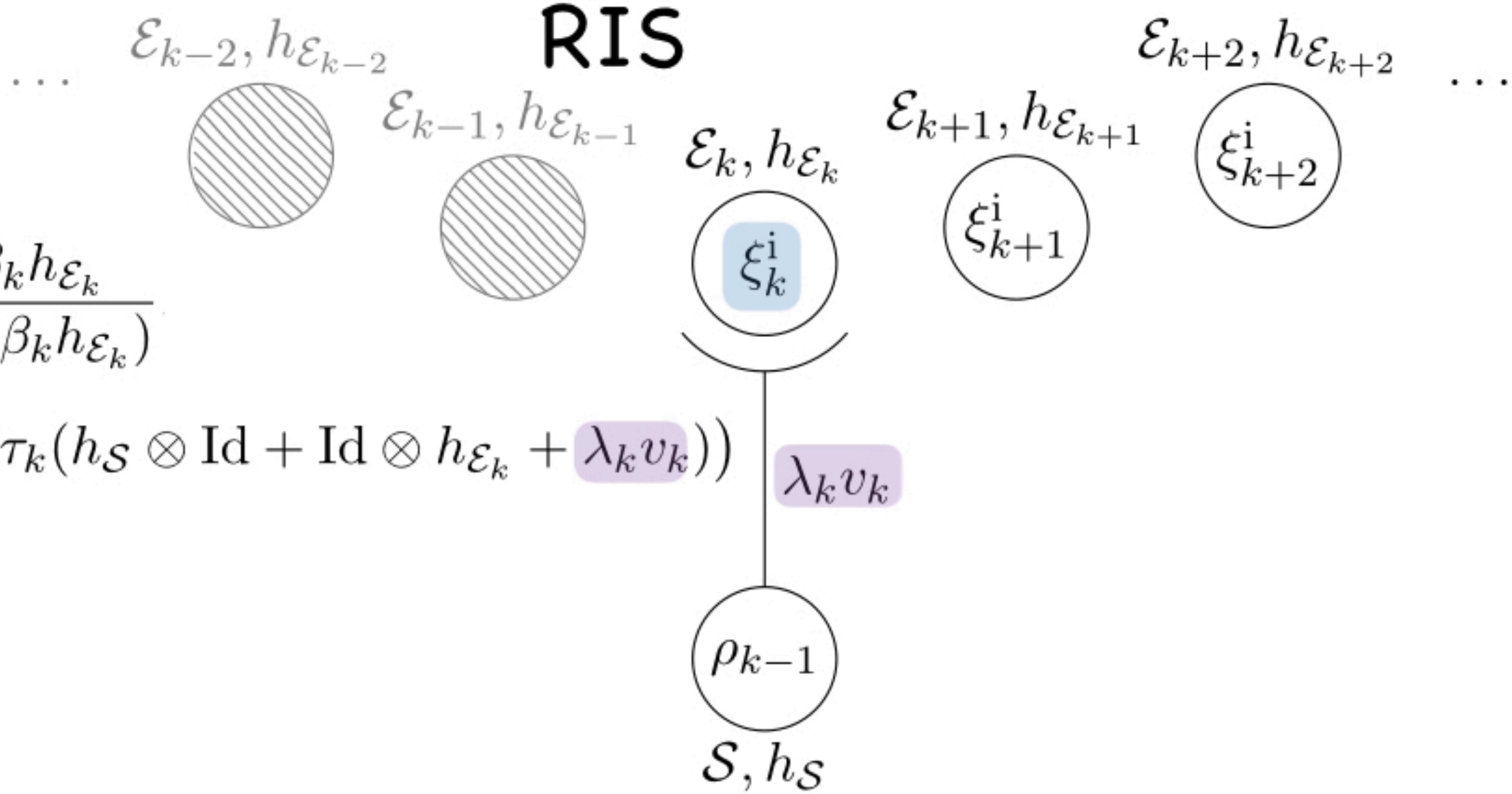

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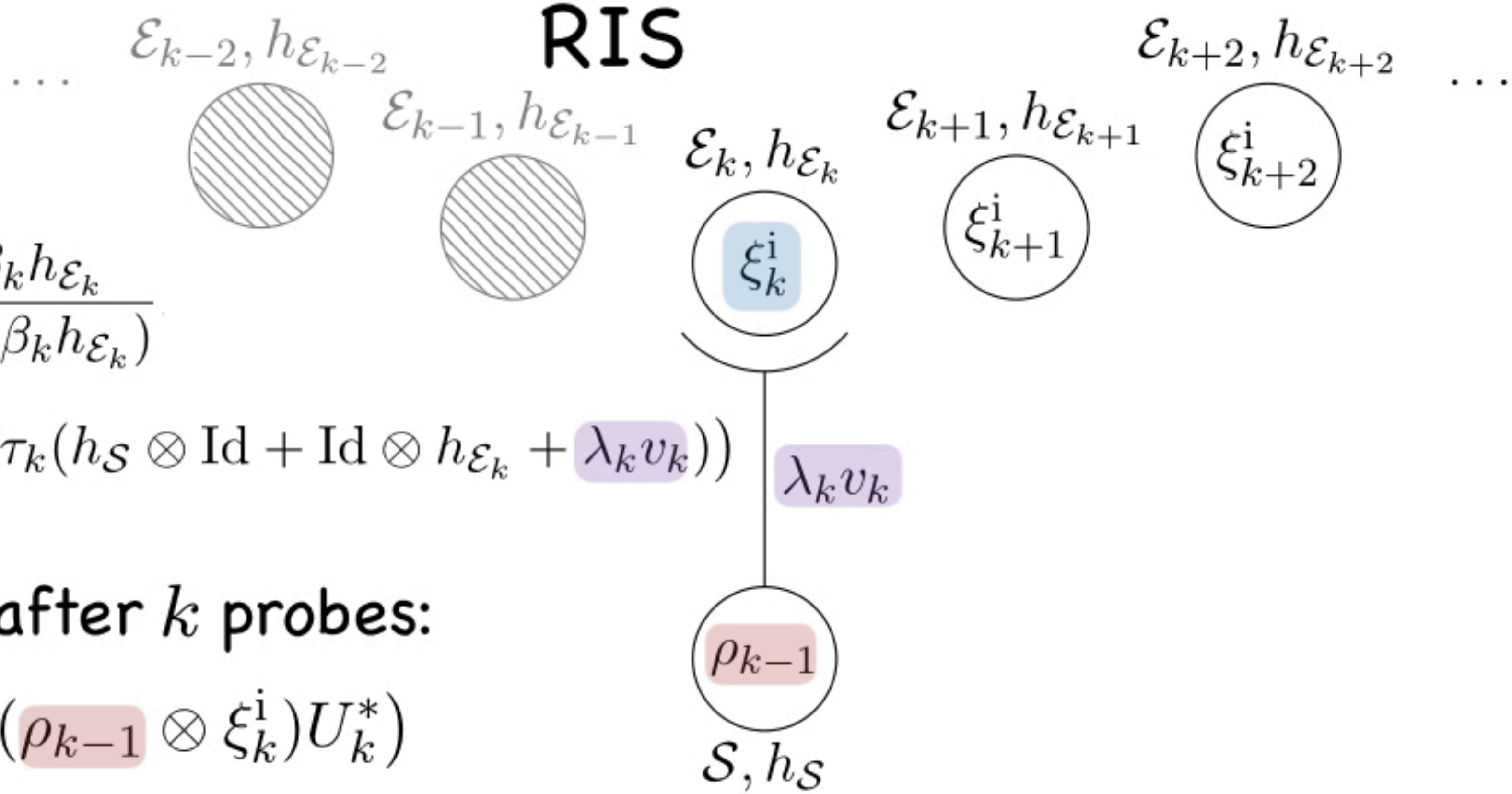
State of $\mathcal{S}$ after k probes:

$$\rho_k = \mathrm{Tr}_{\mathcal{E}_k}\big(U_k(\rho_{k-1} \otimes \xi_k^{\rm i})U_k^*\big)$$



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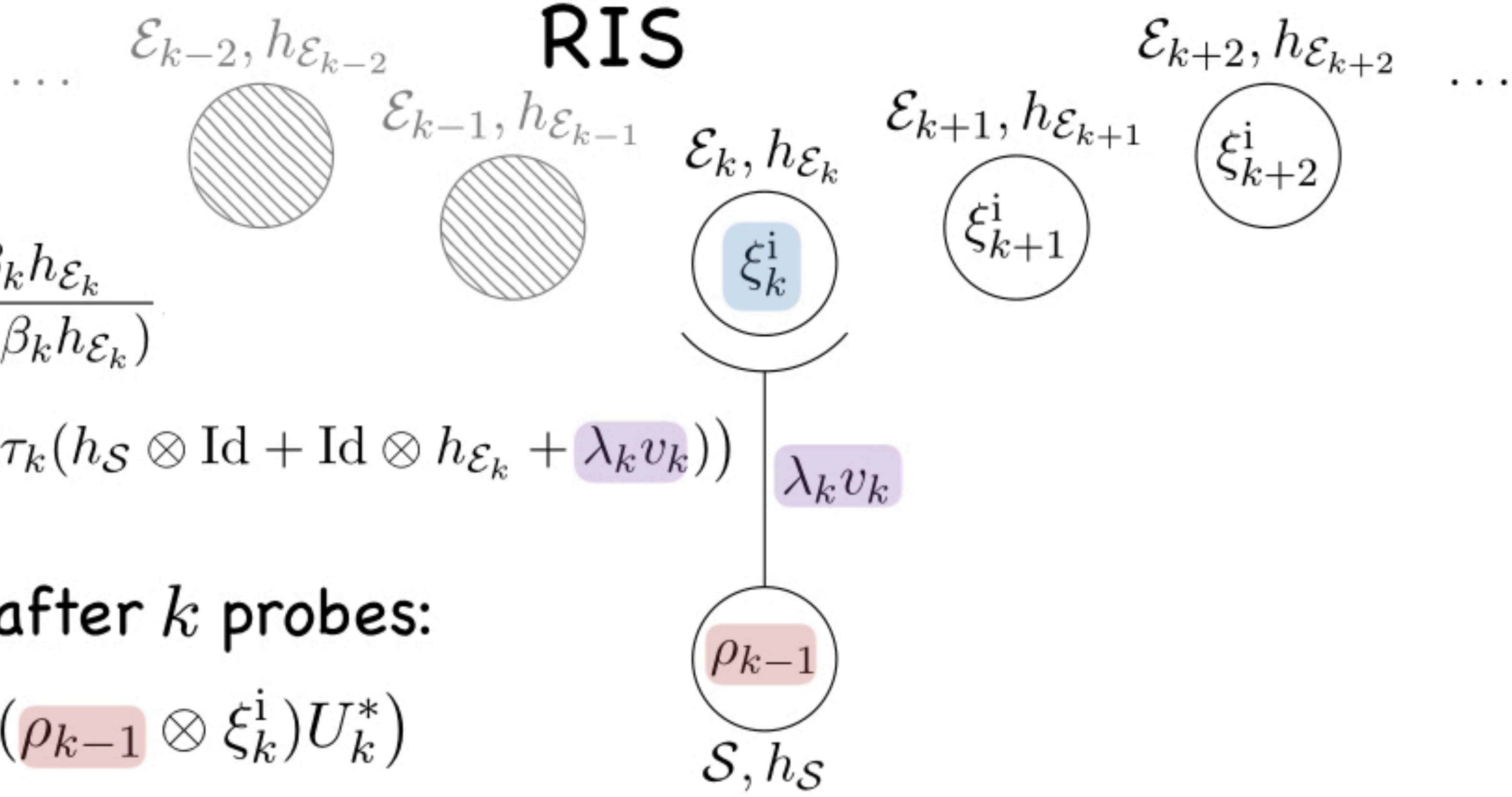

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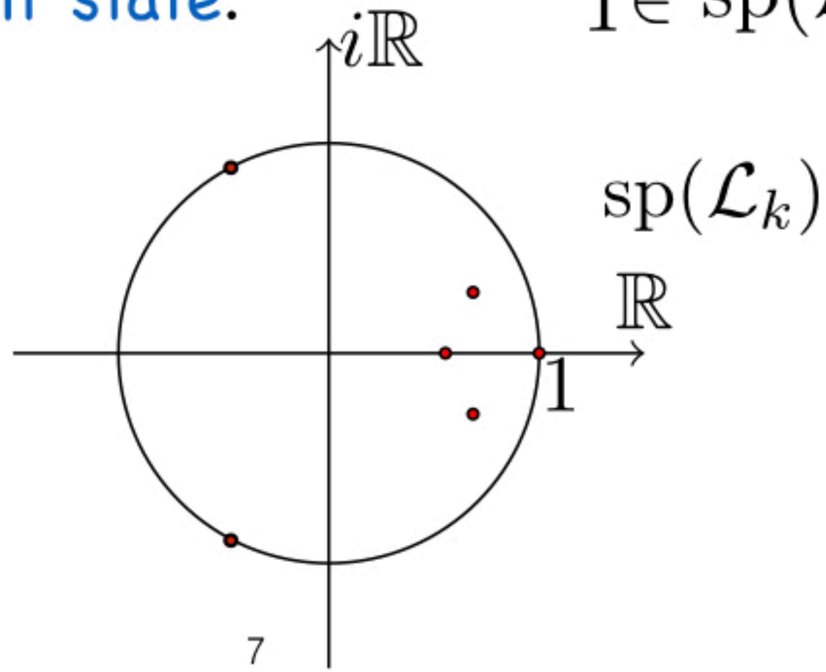
Trace class op. $\mathcal{I}_1(\mathcal{H}_{\mathcal{S}})$

$$\begin{array}{llcl} \mathcal{L}_k : & \mathcal{I}_1(\mathcal{H}_{\mathcal{S}}) & \to & \mathcal{I}_1(\mathcal{H}_{\mathcal{S}}) \\ & \eta & \mapsto & \mathrm{Tr}_{\mathcal{E}_k}\big(U_k(\eta \otimes \xi_k^{\mathrm{i}})U_k^*\big) \end{array}$$

s.t. $\rho_k = \mathcal{L}_k \mathcal{L}_{k-1} \cdots \mathcal{L}_1 \rho^{\mathrm{i}}$

Reduced Dynamics Operator $\mathcal{L}_k$

- Completely positive & trace preserving (CPTP)
- Contraction operator on $\mathcal{I}_1(\mathcal{H}_S)$: $\| \mathcal{L}_k \|_1 \leq 1$
- Admits an invariant state: $1 \in \mathrm{sp}(\mathcal{L}_k)$
- Spectrum:



Landauer's principle for RIS

Step by step

- Entropy balance eq.: $\Delta S_k + \sigma_k = \beta_k \Delta Q_k$

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- First T steps:

$$\sum_{k=1}^{T} \Delta S_k + \underbrace{\sum_{k=1}^{T} \sigma_k}_{:= \sigma_T^{tot}} = \sum_{k=1}^{T} \beta_k \Delta Q_k$$

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- Landauer's bound for RIS:

$$S(\rho^{\mathrm{i}}) - S(\rho_T) \leq \sum_{k=1}^{T} \beta_k \Delta Q_k$$

Adiabatic changes in probes

- Sampling from C^2 functions on $[0,1]$

$$[0,1] \ni s \mapsto \mathcal{L}(s) \in \mathcal{B}(\mathcal{I}_1(\mathcal{H}_\mathcal{S}))$$

$$\mathcal{L}_k = \mathcal{L}_{k,T} := \mathcal{L}(k/T)$$

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- Example: slow variation of temperatures only

$$\beta_{k,T} = \beta\Big(\frac{k}{T}\Big)$$

- Variations are $O(T^{-1})$

Discrete Non-Unitary Adiabatic Thm.

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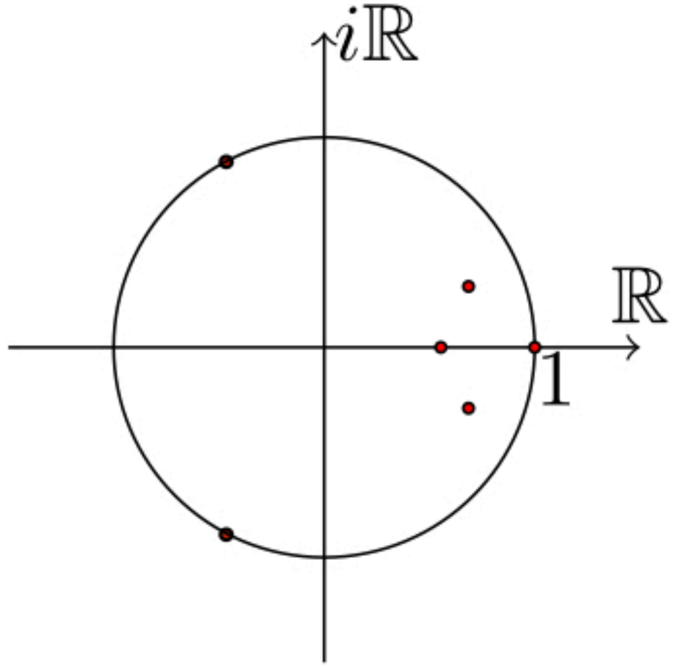

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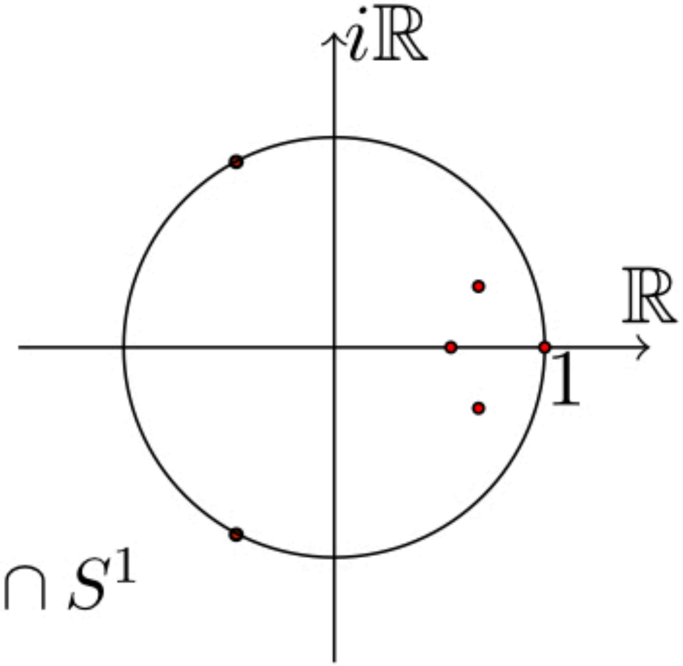

- Simple periph. spectrum, $\mathrm{sp}\,\mathcal{L}(s) \cap S^1$

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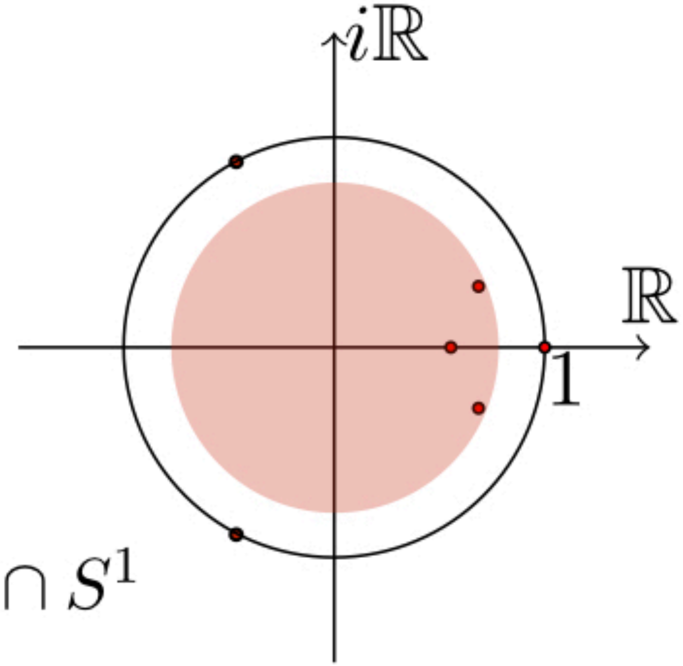

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- With $Q(s) := \mathrm{Id} - P(s)$, $\ell := \sup_{s \in [0,1]} \mathrm{spr}\ \mathcal{L}(s)\, Q(s) < 1$

Ergodicity assumption

Simplif.

- Let $P(s)$ <-> $\{1\} = \mathrm{sp}\,\mathcal{L}(s) \cap S^1$ <-> $\rho_s^{\mathrm{inv}} = P(s)\rho_s^{\mathrm{inv}}$

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and ρ^{i} a faithful state

$$\rho_k = \mathcal{L}_k \cdots \mathcal{L}_1 \rho^{\mathrm{i}} = \rho_{k/T}^{\mathrm{inv}} + O(1/(T(1-\ell)) + \ell^k)$$

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- Rem: control over $\mathcal{L}_k \mathcal{L}_{k-1} \cdots \mathcal{L}_1$ in general

- Unitary discrete — Dranov, et al '98, Tanaka '11

  Non-unitary — Abou-S. et al '05, J. '07, Avron et al '12, Schmidt '14

Two step Measurement Protocol

Horowitz, Parrondo '13

- Assume $h_{\mathcal{E}_k} = \sum_{i=1}^{d} E_i^{(k)} \Pi_i^{(k)}$, $\xi_k = \exp(-\beta_k h_{\mathcal{E}_k})/Z_k$

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- Let $A = \sum_i \mu_i^{(A)} \pi_i^{(A)}$, $B = \sum_i \mu_i^{(B)} \pi_i^{(B)}$ on $\mathcal{S}$

  & $\{h_{\mathcal{E}_k}\}_{k\in\{1,\ldots,T\}}$ on $\mathcal{E}$

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- QM Proba. $\mathbb{P}_F^{(T)}(a, b, \vec{i}, \vec{j})$ of outcomes is

$$\mathrm{Tr}[U_T \cdots U_1 (\pi_a^{(A)} \otimes \Pi_{\vec{i}})(\rho^{\mathrm{i}} \otimes \Xi)(\pi_a^{(A)} \otimes \Pi_{\vec{i}}) U_1^* \cdots U_T^* (\pi_b^{(B)} \otimes \Pi_{\vec{j}})]$$

where $\Xi = \bigotimes_{k=1}^{T} \xi_k$, $\vec{i} = (i_k)_{k=1}^{T}$, $\Pi_{\vec{i}} = \Pi_{i_1}^{(1)} \otimes \Pi_{i_2}^{(2)} \cdots \otimes \Pi_{i_T}^{(T)}$

Fluctuations in Landauer's Principle

- Quantum Trajectory

$$\omega = (a, b, \vec{i}, \vec{j}) \qquad \text{of Proba.} \qquad \mathbb{P}_F^{(T)}(a, b, \vec{i}, \vec{j})$$

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- Let $Y_k = \beta_k h_{\mathcal{E}_k}$, $Y = \sum_{k=1}^{T} Y_k$ & comm. hyp.

Consider

$$\sigma_T(\omega) = \sum_{k=1}^{T} \beta_k (E_{j_k}^{(k)} - E_{i_k}^{(k)}) - \log \frac{\langle b|\rho^{\mathrm{f}} b\rangle}{\langle a|\rho^{\mathrm{i}} a\rangle} \qquad \omega = (a, b, \vec{i}, \vec{j}).$$

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- Fact $\quad \mathbb{E}_{\mathbb{P}_F^{(T)}}(\sigma_T(\omega)) = \sum_{k=1}^{T} \beta_k \Delta Q_k - \big(S(\rho^{\mathrm{i}}) - S(\rho_T)\big) = \sum_{k=1}^{T} \sigma_k$

- Rem: strategy from Benoit, Fraas, Jaksic, Pillet '16

Distorted Reduced Dynamics Operator

- Simplif: $A = \mathbb{I},\ B = \mathbb{I}$ & measure $Y = \sum_{k=1}^{T} Y_k$ only

- RV: Variation $\Delta Y = Y|_T - Y|_0$ $y_{\vec{i}} = \sum_{k=1}^{T} \beta_k \, E_{i_k}^{(k)}$

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- Characteristic Function

$$\phi_T(\alpha) = \mathbb{E}(e^{i\alpha\Delta Y}) = \sum_{\vec{i},\vec{j}} \mathbb{P}_F^{(T)}(\vec{i},\vec{j}) e^{i\alpha(y_{\vec{j}} - y_{\vec{i}})}$$

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- Computation: $\phi_T(\alpha) = \mathrm{tr}_{\mathcal{S}}((\mathcal{L}_{T,T}(\alpha)\circ\cdots\circ\mathcal{L}_{1,T}(\alpha))\rho^{\mathrm{i}})$ where

$$\mathcal{L}_{k,T}(\alpha) : \rho \mapsto \mathrm{tr}_{\mathcal{E}_{k,T}}[U_k(\rho\otimes\xi_k)(\mathbb{I}\otimes e^{-i\alpha Y_k})U_k^*(\mathbb{I}\otimes e^{i\alpha Y_k})]$$

- Rem: Akin to Jaksic, Pillet, Westrich '14

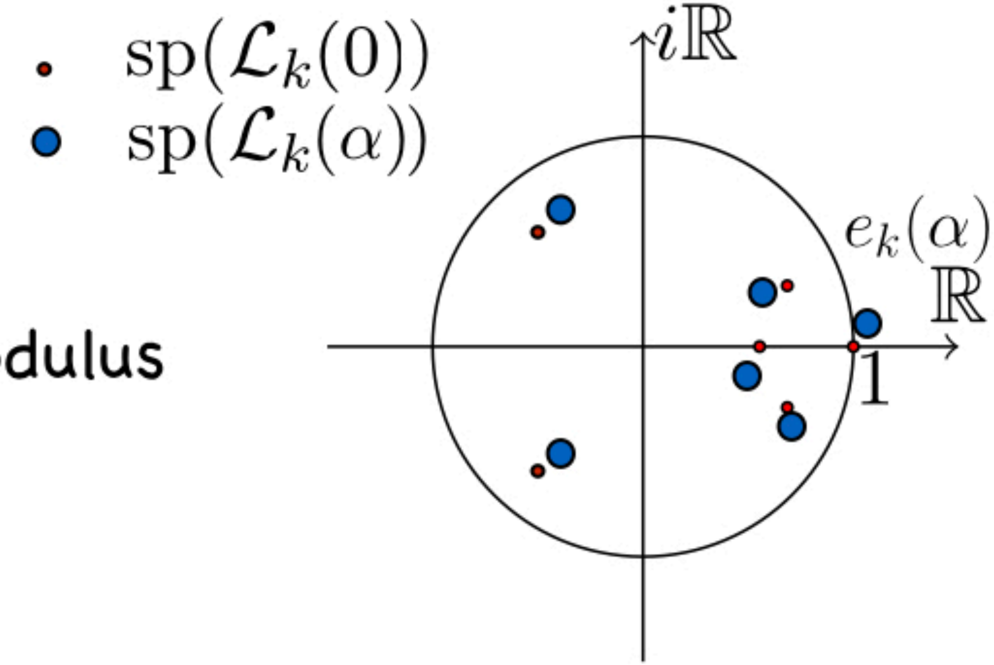
Distorted Reduced Dynamics Operator $\mathcal{L}_k(\alpha)$

- Perturbation:

  $\alpha \in \mathbb{C}$, $|\alpha|$ small

  $e_k(\alpha)$ e.v. of largest modulus

  $e_k(0) = 1$



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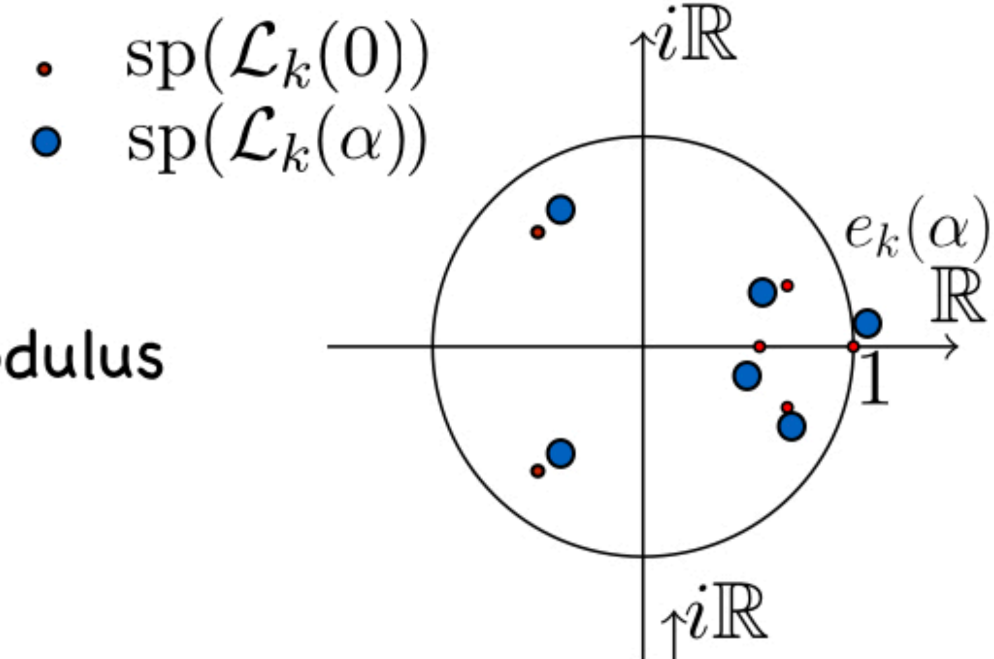
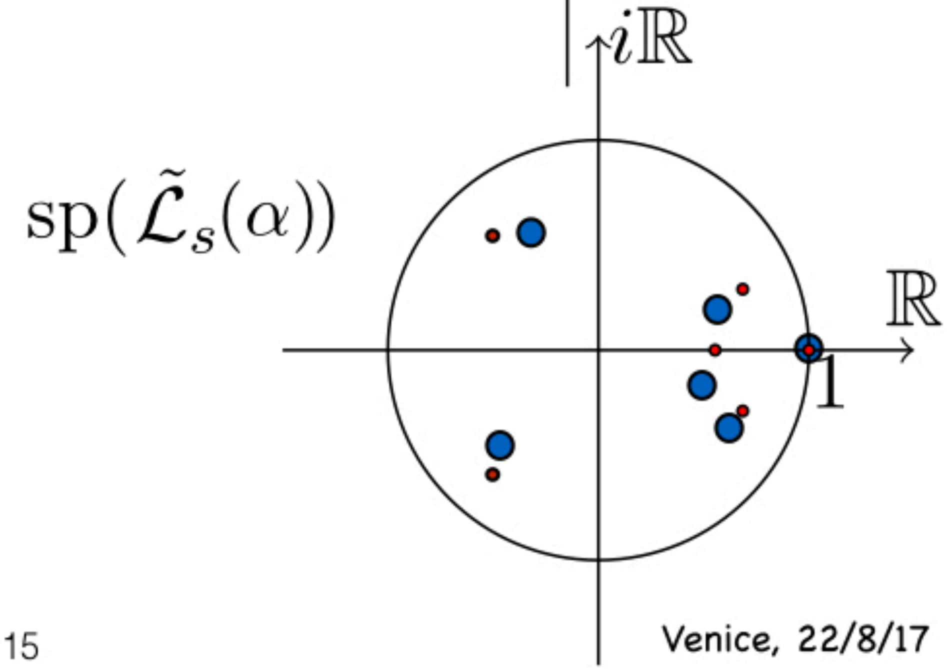


- **Normalisation:**

  $$\tilde{\mathcal{L}}_s(\alpha) := \frac{\mathcal{L}_s(\alpha)}{e_s(\alpha)}$$



Asymptotics of Characteristic Function

$\alpha \in \mathbb{C}$, $|\alpha|$ small with $\tilde{\mathcal{L}}_s(\alpha) := \frac{\mathcal{L}_s(\alpha)}{e_s(\alpha)}$

$$\frac{1}{T} \log \phi_T(\alpha) = \frac{1}{T} \sum_{k=1}^{T} \log e_k(\alpha) + \frac{1}{T} \log \mathrm{Tr}[\tilde{\mathcal{L}}_{T,T}(\alpha) \cdots \tilde{\mathcal{L}}_{1,T}(\alpha) \rho^{\mathrm{i}}]$$

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HJPR '17

- Law of Large Numbers $\mathbb{E}_{\mathbb{P}_F^{(T)}}\big(\sigma_T(\omega)\big) \simeq A_1 T,\ A_1 \geq 0$

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- Central Limit Theorem $\frac{\sigma_T(\omega)-A_1T}{\sqrt{T}} \to \mathcal{N}(0, A_2)$

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- Central Limit Theorem $\frac{\sigma_T(\omega) - A_1 T}{\sqrt{T}} \to \mathcal{N}(0, A_2)$
- Large Deviations Principle beyond perturb. regime

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$\alpha \in \mathbb{C}$, $|\alpha|$ small with $\tilde{\mathcal{L}}_s(\alpha) := \frac{\mathcal{L}_s(\alpha)}{e_s(\alpha)}$

$$\frac{1}{T}\log\phi_T(\alpha) = \frac{1}{T}\sum_{k=1}^{T}\log e_k(\alpha) + \underbrace{\frac{1}{T}\log \mathrm{Tr}[\tilde{\mathcal{L}}_{T,T}(\alpha)\cdots\tilde{\mathcal{L}}_{1,T}(\alpha)\rho^{\mathrm{i}}]}_{\text{adiab. thm}}$$

$$\xrightarrow{T\to\infty} \int_0^1 \log e_s(\alpha)\,\mathrm{d}s$$

HJPR '17

- Law of Large Numbers $\mathbb{E}_{\mathbb{P}_F^{(T)}}\big(\sigma_T(\omega)\big) \simeq A_1 T,\ A_1 \geq 0$

- Central Limit Theorem $\frac{\sigma_T(\omega)-A_1T}{\sqrt{T}} \to \mathcal{N}(0, A_2)$

- Large Deviations Principle beyond perturb. regime

- Rem: Benoit, et al. '16 get discrete RV