Quantum Dynamics of Systems Under Repeated Observation

Reconstruction of Structure from Unstructured Perception

Jürg Fröhlich (ETH Zurich)

Spring, 2017

Outline

We start by presenting a short summary of examples of *"effective dynamics" in quantum theory*.

We then study more closely the effective quantum dynamics of systems interacting with a long chain of independent probes, one after another, which, afterwards, are subject to a projective measurement and are then lost.

This leads us to develop a theory of indirect measurements of time-independent quantities (non-demolition measurements).

Next, the theory of indirect measurements of time-dependent quantities is outlined, and a new family of diffusion processes – quantum jump processes – is described.

Some open problems are proposed.

In memory of my friend the late Claude Itzykson

Credits and Contents

- *Thanks are due to:* B. Schubnel, M. Ballesteros, Ph. Blanchard, W. De Roeck, N. Crawford and M. Fraas – for sometimes delightful collaborations; and to M. Bauer, D. Bernard, D.-A. Deckert, D. Dürr, B. Kümmerer, H. Maassen, S. Teufel, A. Tilloy, and others – for useful discussions.

- *Contents:*
  1. Examples of effective (quantum) dynamics
  2. Systems subject to repeated observation – Haroche-Raimond- and solid-state experiments
  3. Indirect non-demolition measurements: General results
  4. Weak measurements of time-dependent quantities - Markov chains on spectra of observables
  5. Open problems, conclusions

1. Examples of effective (quantum) dynamics

Here is a list of examples of effective quantum dynamics that are of obvious physical interest and quite non-trivial to analyze:

- Dynamics in the mean-field regime: Very high density of particles, very weak two-body interactions; (first studied by Klaus Hepp). The mean-field limit is a classical (field-, or continuum-) limit of QTh, and one can use, e.g., Egorov-type theorems to analyze it. It is the converse of the process of quantizing continuum theories of matter, such as the Vlasov- and the Hartree equations; $\nearrow$ "atomism as quantization". – Other regimes: Gross-Pit. lim, ...
- Particle limit of continuum theories: E.g., Hartee solitons as point-like particles exhibiting damped Newtonian motion – possibly interesting in cosmology!
- Kinetic- or Van Hove regime: Weak interaction of a "small" system with an infinite (thermal) reservoir; (time rescaled by inverse square of coupling constant) $\rightarrow$ "Return to Equilibrium", Approach to a NESS, etc.
  Mathematical methods: Singular perturbation theory, e.g., in the form of the BFS Feshbach-RG

Effective dynamics - ctd.

- Isothermal processes: Quasi-static motion of "small" system coupled to a thermostat – isothermal theorem $\simeq$ adiabatic theorem.
- Relaxation to Ground-States & (Quantum) Brownian Motion: "Small" system coupled to ∞-extended quantized harmonic wave medium at $T = 0$ relaxes to its *ground-state*; (F-Gr-Schl, DeR-K). A *particle with internal degrees of freedom cpld. to modes of harmonic thermostat* (consisting of, e.g., an ideal NR Bose gas) moving in $\mathbb{Z}^d, d \geq 3$,exhibits diffusive motion $\rightarrow$ "QBM"! For highly simplified models, *Einstein relation* betw. particle mobility and diffusion const. can be established; (see DeR-F-Schn). With disorder: Thermal noise always destroys localization; (see F-Sche). $\leftarrow$ Expansions around kinetic lim, using (many-scale) cluster exp..
- Motion with friction: Particle coupled to a wave medium, such as em field in an optically dense medium, or sound waves in a B-E condensate, emits Cherenkov radiation, causing deceleration of its motion until speed is $\leq$ speed of wave propagation in medium. Analyzed in mean-field- (FG-Z) and kinetic limit (B-DeR-F). Spectral th.: DeR-F-Pizzo.

Fundamental quantum dynamics of physical systems

- Dynamics of systems featuring events – *"ETH in QM":*
  Fundamental problems concerning *Foundations of Quantum Mechanics* are encountered when one studies the notion of *"events"* in QM and the question of how events can be recorded, using *"instruments"* – viz. the theory of *projective measurements*. I have undertaken a considerable effort to elucidate problems surrounding events and projective measurements (of events).[1] My results give rise to the *"ETH approach to quantum mechanics"* – for:

  "Events, Trees, and Histories".

  $\Rightarrow$ Fundamental qm dynamics of states of phys. systems featuring events can be described in terms of a new kind of *stochastic branching process* whose (non-commutative) state space can be described in terms of families of orthogonal projections. In NR quantum mechanics, branchings are labelled by time and happen continuously. – The fundamental principle underlying the *ETH approach* is the *"Principle of Loss of Access to Information"*.

[1] I'd be happy to talk about my results, but cannot present them here.

2. Systems subject to repeated observation – Haroche-Raimond- and solid-state experiments

The *ETH approach* represents a *"quantum theory without observers"*. In comparison to the conceptually subtle theory of projective measurements, the theory of *indirect* (in particular, non-demolition-) *measurements* is fairly straightforward and can be presented with mathematical precision.

The general *Theory of Indirect Measurements* of physical quantities – pioneered by Karl Kraus – is the *main topic of this lecture.*

Karl Kraus (1938-1988)

A metaphor for the theory of indirect observations

Plato's Allegory of the Cave – *'Politeia'*, in: Plato's *'Republic'*

As Plato was anticipating, more than 350 years BC, all we "prisoners of our senses" are able to perceive of the world are "shadows of reality", in the form of long streams of crude, uninteresting, directly perceptible signals (= "projective measurements") from which well structured, meaningful facts and events can be reconstructed. Socrates explains:

philosophers = mathematicians and theoretical physicists

are "liberated prisoners" who are able to reconstruct the fabric of reality from the shadows it creates on the wall of the cave.

Systems/experiments to be studied

Sketch of the Haroche-Raimond experiment:

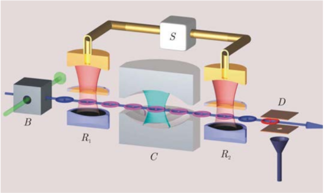
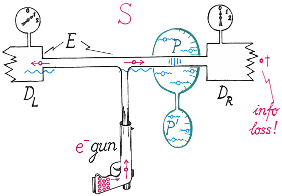


Fig. 4: Experimental setup to study microwave field states with the help of circular Rydberg atoms (see text).

B: atom gun, R_1: State prep., C: Cavity, R_2: ..., D: Detector

Sketch of a putative solid-state experiment:



Capture of Sketches

Isolated system $S := E \vee P$, where $P =$ cavity C/quantum dot, $E =$ "environment/equipment" consisting of:

(1) Probes: Independent atoms $A_1, A_2, \dots$ prepared in R_1/ indep. e^- traveling through $T-$shaped wires. During time interval $[(m-1)\tau, m\tau)$, m^{th} atom streams through cavity/m^{th} e^- travels from e^--gun to one of the two detectors D_L, D_R; $\tau =$ duration of a measurement cycle.

(2) an atom detector D/two electron detectors D_L, D_R.

It is a little easier to picture how the *solid-state experiment* works: Observables referring to quantum dot P:

$$\mathcal{O}_P := \{\text{functions of } e^-\text{-number operator } \mathcal{N}\}$$

Observables referring to E:

$$\mathcal{O}_E = \{\mathbf{1}_P \otimes \mathbf{1}_{e_1^-} \otimes \cdots \otimes X_{e_m^-} \otimes \mathbf{1}_{e_{m+1}^-} \otimes \dots\}_{m=1,2,3,\dots},$$

Description of solid-state experiment

where the operator $X_{e_m^-}$ acts on the one-particle Hilbert space of the m^{th} electron traveling through the $T-$ shaped wires towards D_L, D_R, resp. It is given by

$$X_{e_m^-} = \begin{pmatrix} \mathbf{1} & 0 \\ 0 & -\mathbf{1} \end{pmatrix},$$

with infinitely degenerate eigenvalues $\xi = \pm 1$:

$$\xi = +1 \leftrightarrow e_m^- \text{ hits } D_L, \qquad \xi = -1 \leftrightarrow e_m^- \text{ hits } D_R.$$

From now on, "L" is replaced by $+1$ and "R" by -1. The eigenprojection of $X_{e_m^-}$ corresponding to the eigenvalue ξ is denoted by π_ξ^m; $X_{e_m^-}$ is measured around time $m\tau$.
Let ρ denote the state of S. Our aim is to determine the probability of the events that, for $j = 1, 2, \ldots, m$, the j^{th} electron hits the detector D_{ξ_j}; $\quad m = 1, 2, 3, \ldots$.

The LSW formula

For (strictly) independent electrons [2], this probability is given by a formula proposed by Lüders, Schwinger and Wigner (LSW):

$$\mu_\rho\big(\xi_1, \xi_2, \ldots, \xi_m\big) = \mathrm{tr}\big(\pi^m_{\xi_m} \cdots \pi^1_{\xi_1}\, \rho\, \pi^1_{\xi_1} \cdots \pi^m_{\xi_m}\big) \tag{1}$$

Since $\pi^m_1 + \pi^m_{-1} = \mathbf{1}, \forall m$, and because of cyclicity of the trace,

$$\sum_{\xi_m} \mu_\rho(\xi_1, \xi_2, \ldots, \xi_{m-1}, \xi_m) = \mu_\rho(\xi_1, \xi_2, \ldots, \xi_{m-1})$$

Thus, by a lemma due to Kolmogorov, μ_ρ extends to a measure on the space, Ξ, of "histories" ($= \infty$ long measurement protocols $\underline{\xi} = \big(\xi_j\big)_{j=1}^{\infty}$), equipped with σ-algebra, Σ, generated by cylinder sets. First, consider the situation where the passage of e^-'s from the electron gun through the $T-$ shaped wire to one of the detectors $D_\xi, \xi = \pm 1$, does not affect the charge, ν, of the quantum dot P, which is a conserved quantity $\rightarrow$ *"non-demolition measurements"*.

[2] the property of strict indep. of e^-'s is a special case of *"decoherence"*

Non-demolition measurement & exchangeable probabilities

Then one can argue that the measure μ_ρ is *exchangeable*:

$$\mu_\rho\big(\xi_{\sigma(1)}, \dots, \xi_{\sigma(m)}\big) = \mu_\rho\big(\xi_1, \dots, \xi_m\big), \tag{2}$$

for all permutations, σ, of $\{1, \dots, m\}$, for arbitrary $m < \infty$. It then follows from De Finetti's Theorem that

$$\mu_\rho(\xi_1, \dots, \xi_m) = \int_{\Xi_\infty} \mathrm{d}P_\rho(\nu) \prod_{j=1}^{m} p(\xi_j|\nu) \tag{3}$$

Here Ξ_∞ is the spectrum of the algebra of bounded measurable functions on Ξ that are measurable at ∞[3]. Ξ_∞ is the "space of facts", or "Dinge an sich", i.e., the true reality Plato is talking about, whereas the measurement protocols $\underline{\xi}_m := (\xi_j)_{j=1}^m, m < \infty$, are the shadows on the wall of the cave that the prisoners are able to perceive, as we shall now explain!

---

[3] equivalence classes w.r. to a measure class (determined by normal states of S) of functions on Ξ not depending on any finite nb. of measurement outcomes

Interpretation of Ξ_∞ in the solid-state experiment

Suppose that every electron traveling from the e^--gun to one of the detectors $D_{\pm 1}$ is prepared in the same one-particle state ϕ_0. Assuming that the charge operator, $\mathcal{N}$, of the quantum dot P is a conservation law, the time evolution of the state ϕ_0 during one measnt. cycle is given by

$$U_\nu \phi_0,$$

where U_ν is a unitary operator on the one-electron Hilbert space depending on the charge ν of P: The charge ($\propto$ nb. of e^-) bound by P creates a "Coulomb blockade" in the right arm of the $T-$ shaped wire; whence the larger ν, the more likely it is that an electron in the wire will be scattered onto the detector $D_1 \equiv D_L$.

The projection of one-electron wave functions that vanish identically near $D_{-\xi}$ is denoted by π_ξ. The probability, $p(\xi|\nu)$, that an e^- hits D_ξ is given by *Born's Rule*

$$p(\xi|\nu) = \langle \phi_0, U_\nu^* \pi_\xi U_\nu \, \phi_0 \rangle, \qquad (4)$$

and the space Ξ_∞ of "Dinge an sich" is given by

$$\Xi_\infty = \text{spec}(\mathcal{N}) = \{0, 1, 2, \ldots, N\}, N < \infty, \quad \mathcal{N} = \text{charge operator of } P.$$

3. Indirect Non-Demolition Measurements: General Results

Let us consider a slightly more general context: $\mathcal{X}_S$ is the space of possible outcomes of probe measurements, (with $\mathcal{X}_S = \{-1, +1\}$, in the solid state experiment), and let $\Xi = \mathcal{X}_S^{\times \mathbb{N}}$ be the "space of histories". Assuming *"decoherence"* for consecutive probe measurements, the measures μ_ρ on Ξ can be decomposed over the spect., Ξ_∞, of equivalence classes of functions measureable at ∞:

$$\mu_\rho(\underline{\xi}) = \int_{\Xi_\infty} \mathrm{d}P_\rho(\nu)\, \mu(\underline{\xi}|\nu), \tag{5}$$

where the measures $\mu(\cdot|\nu)$ are mutually singular, and $P_\rho(\Delta)$ is the *Born probability* of observing a "fact" belonging to the set $\Delta \in \Xi_\infty$, given that the system S has been prepared in state ρ. Actually, (assuming *"asymptotic abelianess"*) the measures $\mu(\cdot|\nu)$ come from normal states of S, and the "space of facts" Ξ_∞ can be shown to be contained in or equal to the spectrum of an algebra, $\mathcal{E}_\infty$, of operators at time $t = \infty$ in the center of the algebra of "observables" of S; (BFFS).

Basic assumptions

As in the example of the solid-state experiment, we will henceforth assume[4] that:

(i) The measures μ_ρ are exchangeable (non-demolition observations), so that

$$\mu(\underline{\xi}_m|\nu) = \prod_{j=1}^{m} p(\xi_j|\nu).$$

(ii) The space of "facts" is a finite set of points ("charge values")

$$\Xi_\infty = \{0, 1, 2, \dots, N\}, \qquad \text{for some } N < \infty. \tag{6}$$

(iii) We also assume that $p(\xi|\cdot)$ separates points of Ξ_∞: There exists $\kappa > 0$ such that

$$\min_{\nu_1 \neq \nu_2} |p(\xi|\nu_1) - p(\xi|\nu_2)| \geq \kappa > 0, \quad \text{for some } \xi \in \mathcal{X}_S. \tag{7}$$

[4] these assumptions can and have been generalized

Summary of main results

Equivalence classes of functions on the space Ξ of histories meas. at ∞ form an abelian algebra isomorphic to the algebra of "observables at infinity" (= funs. on the "space of facts" Ξ_∞), which is isomorphic to $\text{Diag}_{(N+1)}$. An example of an "observable at infinity" is the *"asymptotic frequency"* of an event $\xi \in \mathcal{X}_S$: We define the frequencies

$$f_\xi^{(l,l+k)}(\underline{\xi}) := \frac{1}{k}\left(\sum_{j=l+1}^{l+k} \delta_{\xi,\xi_j}\right), \quad \text{with } \sum_\xi f_\xi^{(l,l+k)}(\underline{\xi}) = 1. \tag{8}$$

Main results:

(1) *Law of Large Numbers for exchangeable measures:* The asymptotic frequency satisfies

$$\lim_{k\to\infty} f_\xi^{(l,l+k)}(\underline{\xi}) =: p(\xi|\nu), \tag{9}$$

for some point (or "fact") $\nu \in \Xi_\infty$. (Special case: Experiments explained at the beginning.)

“q-hypothesis testing” /parameter estimation

With each $\nu \in \Xi_\infty$ we associate a subset

$$\Xi_\nu(l,k;\underline{\varepsilon}) := \{\underline{\xi} \mid |f_{\underline{\xi}}^{(l,l+k)}(\underline{\xi}) - p(\xi|\nu)| < \epsilon_k\}, \tag{10}$$

where

$$\epsilon_k \to 0, \sqrt{k}\,\epsilon_k \to \infty, \quad \text{as } k \to \infty$$

(2) *Distinguishability:* It follows from Hyp. (7) and definition (8) that, for k so large that $\epsilon_k < \kappa/2$,

$$\Xi_{\nu_1}(l,k;\underline{\varepsilon}) \cap \Xi_{\nu_2}(l,k;\underline{\varepsilon}) = \emptyset, \quad \nu_1 \neq \nu_2.$$

(3) *Central Limit Theorem:* $\Rightarrow$ Under suitable hypotheses on the states ρ, e.g., (i) through (iii),

$$\mu_\rho\left(\bigcup_\nu \Xi_\nu(l,k;\underline{\varepsilon})\right) \to 1, \quad \text{as } k \to \infty.$$

(1), (2) & (3) $\Rightarrow$ As $m \to \infty$, every history $\underline{\xi}_m$ of measurement outcomes determines a unique point (“charge”) $\nu \in \Xi_\infty$; (error $\to 0$, as $m \to \infty$).

hypothesis testing – ctd.

Moreover, *Born's Rule* holds: $\mu_\rho\big(\Xi_\nu(l,k;\underline{\varepsilon}\big) \to P_\rho(\nu),\ k \to \infty$.

(4) *Theorem of Boltzmann-Sanov* $\Rightarrow$ If the measures μ_ρ are exchangeable one has that

$$\mu\big(\Xi_{\nu_1}(l,k;\underline{\varepsilon})|\nu_2\big) \le C\ e^{-k\sigma(\nu_1\|\nu_2)}$$

where σ is the relative entropy of the distribution $p(\cdot|\nu_1)$ given $p(\cdot|\nu_2)$

.

(5) *Theorem of Maassen and Kümmerer* $\cdots \Rightarrow$ In the Haroche-Raimond experiment described above, the state of S, restricted to $B(\mathcal{H}_P)$, approaches a state, ρ^ν, with a *fixed number, ν, of photons* in the cavity $P(\equiv C)$, as $k \to \infty$: "Purification"!

(Analogous results for solid-state experiment.)

The theory of indirect measurements outlined here only concerns measurements of time-independent "facts", which correspond to points in Ξ_∞ (*non-demolition measurements!*). However, most interesting facts depend on time, i.e., are "events", and $\Xi_\infty = \emptyset$! Thus, we must ask how one can acquire information concerning *events* indirectly, through repeated direct measurements of quantities corresp. to operators in $\mathcal{O}_E$.

4. Weak Measurements of Time-Dependent Quantities –

Markov Jump Processes on Spectra of Observables & Mott Tracks

We consider an isolated physical system $S = P \vee E$, as before.
States of S are given by density matrices, ρ_S, acting on a Hilbert space $\mathcal{H}_S = \mathcal{H}_P \otimes \mathcal{H}_E$, where $\mathcal{H}_P = \mathbb{C}^{N+1}$, for some $N < \infty$. When restricted to observables of P, states are given by density matrices

$$\rho_P := \text{tr}_E \, \rho_S. \tag{11}$$

Hilbert space of a single probe A_j: $\mathcal{H}_{A_j} \simeq \mathcal{H}_A$
Initial state of each probe A_j: $\phi_0 \in \mathcal{H}_A$.
Reference state in $\mathcal{H}_E$: $\bigotimes_{j=1}^{\infty} \phi_0^{(j)}, \quad \phi_0^{(j)} = \phi_0, \forall j$.
Space $\mathcal{H}_E =$ completion of linear span of vectors $\quad \bigotimes_{j=1}^{\infty} \psi^{(j)}$,
with $\psi^{(j)} = \phi_0$, except for finitely many j.
For each probe A_j, the same observable, represented by the operator

$$X = \sum_{\xi \in \mathcal{X}_S} \xi \pi_\xi, \qquad \text{card}\,(\mathcal{X}_S) = k < \infty, \tag{12}$$

acting on $\mathcal{H}_{A_j}$, is measured in a detector D.

The formalism

(D will not play any role in the following, hence is omitted.) During the j^{th} measurement cycle $(t_{j-1}, t_j]$, *only* A_j interacts with P, at time t_j. Measurement results for probes $A_1, \dots, A_{j-1} : \underline{\xi}_{j-1} = (\xi_k)_{k=1}^{j-1}$.

Some notations:

- $\rho_P^{(j-1)} \equiv \rho^{(j-1)}(\underline{t}_{j-1}, \underline{\xi}_{j-1})$, $\underline{t}_{j-1} := (t_k)_{k=1}^{j-1}$: State of P right after interaction with A_{j-1}, at time t_{j-1}.
- Let $\mathcal{N}$ be a "charge operator" acting on $\mathcal{H}_P$ with simple spec$(\mathcal{N}) = \{0, 1, \dots, N\}, N < \infty$: E_ν: spectral projection of $\mathcal{N}$ corresp. to ev ν.

Time evolution of $P \vee A_j$ from time t_{j-1} to time t_j right before A_j is subject to projective measurement of X with outcome ξ_j, is given by:

$$\rho_{P\vee A_j} := \sum_{\nu,\nu'} E_\nu e^{-i(t_j - t_{j-1})H_P} \rho_P^{(j-1)} e^{i(t_j - t_{j-1})H_P} E_{\nu'} \otimes U_\nu |\phi_0\rangle\langle\phi_0| U^*_{\nu'}, \quad (13)$$

where H_P is the Hamiltonian of P, and U_ν is a unitary on $\mathcal{H}_A$ mapping the initial state, ϕ_0, of A_j onto the state of A_j right *after* its interaction with P, given that the charge of P at time t_j is given by ν. Then the observable X is measured projectively for A_j, with result $\xi_j \in \mathcal{X}_S$.

Formalism – ctd.

This yields a recursion formula for the state $\rho_P^{(j)}$:

$$\rho_P^{(j)} = \mathcal{Z}_{\xi_j}^{-1} V_{\xi_j} e^{-i(t_j - t_{j-1})H_P} \, \rho_P^{(j-1)} \, e^{i(t_j - t_{j-1})H_P} V_{\xi_j}, \qquad (14)$$

where $\mathcal{Z}_\xi$ is a normalization factor, and V_ξ is given by

$$V_\xi = \sum_\nu V_\xi(\nu), \text{ where } V_\xi(\nu) := E_\nu \sqrt{p(\xi|\nu)}$$

with $p(\xi|\nu) := \langle U_\nu \phi_0, \Pi_\xi \, U_\nu \phi_0 \rangle$; ($\nearrow$ (iii),Sect. 3). Note that

$$V_\xi = V_\xi^*, \; [V_\xi, \mathcal{N}] = 0, \; \forall \xi, \quad \text{and} \quad \sum_{\xi' \in \mathcal{X}_S} V_{\xi'}^2 = \mathbf{1}. \qquad (15)$$

The recursion formula (14) yields a trajectory of states of the subsystem P (the cavity/quantum dot) given by

$$\boxed{\rho_t(\underline{t}_j, \underline{\xi}_j) := e^{-i(t-t_j)H_P} \rho_P^{(j)}(\underline{t}_j, \underline{\xi}_j) e^{i(t-t_j)H_P}, \; t_j < t < t_{j+1}.} \qquad (16)$$

Averaged time-evolution of state of P

We now suppose that the times t_j of interaction between the probes A_j and the subsystem P are Poisson distributed, with rate $\gamma = 1, \forall j$. Fixing a time t and taking an average over measurement times and measurement outcomes, we find that

$$\mathbb{E}\big[\rho_t(\underline{t}_{\cdot}, \underline{\xi}_{\cdot})\big] = e^{t\,\mathcal{L}}\,\rho, \tag{17}$$

where ρ is the initial state of the subsystem P (at time $t = 0$), and $\mathcal{L}$ is a Lindblad generator given by

$$\mathcal{L}\rho = -i\,\mathrm{ad}_{H_P}(\rho) + \Big(\sum_{\xi \in \mathcal{X}_S} V_\xi\,\rho\,V_\xi\Big) - \rho. \tag{18}$$

Eq. (16) is called *"unravelling"* of the Lindblad evolution (17); it appears as the integrand in the Dyson expansion of the right side of (17), with the second term on the right side of (18) treated as the perturbation.

Main result

We suppose that the "Basic Assumptions", (i)-(iii), of Sect. 3 are valid. We assume furthermore that

$$H_P = \varepsilon h_p, \qquad \text{for some } \varepsilon > 0, \tag{19}$$

and we rescale time: $t = \varepsilon^{-2}\tau$. We define a continuous-time Markov jump process, with state space = spec($\mathcal{N}$), paths $\nu_\tau(\omega)$, $\omega = (\underline{t}, \underline{\xi})$, and transition function generated by the Markov kernel:

$$Q(\nu, \nu') = -\frac{|\langle \nu | h_P | \nu' \rangle|^2}{\sum_{\xi \in \mathcal{X}_S} V_\xi(\nu) V_\xi(\nu') - 1} + cc, \quad \nu \neq \nu',$$

with $Q(\nu, \nu) = \cdots \geq 0, \forall \nu$.

We are now prepared to state our main result, (which is a "baby version" of the *"ETH approach"* to QM!).

Main result – ctd.

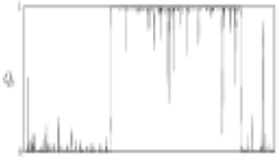
Theorem.

Convergence of qm evolution to Markov jump process:

$$\lim_{\varepsilon \searrow 0} \mathbb{E}\left[\rho_{\varepsilon^{-2}\tau}\left(\omega = (\underline{t}, \underline{\xi})\right)\right] = e^{-\tau Q} \rho_0,$$

where $\rho_0 = \text{Diag}\left(\langle \nu | \rho | \nu \rangle\right)$;
The state $\rho_{\varepsilon^{-2}\tau}\left(\omega = (\underline{t}, \underline{\xi})\right)$ approaches in law a diagonal matrix, $\text{Diag}\left(\delta_{\nu, \nu_\tau(\omega)}\right)$.

Numerical simulation for the behaviour of the diagonal matrix elements of $\rho_{\varepsilon^{-2}\tau}(\underline{t}, \underline{\xi})$ in the special case where $N = 1$ (i.e., $\mathcal{H}_P = \mathbb{C}^2$), for small ε:



5. Open Problems, Conclusions

- More general models of probes and "cavities"; in particular:
- Weakly correlated probes; infinite-dimensional state spaces for cavity, P; operators $\mathcal{N}$ with continuous spectrum (!); ...
- More general models of indirect measurements of time-dependent quantities. –
  Important Example – to be carried out more fully: Consider observables, $\vec{\mathcal{N}}$, with continuous spectrum, $\sigma(\vec{\mathcal{N}}) \simeq \mathbb{R}^d$. Then H_P may generate dynamics describing inertial motion on $\sigma(\vec{\mathcal{N}})$, and the full dynamics of P then describes tracks on $\sigma(\vec{\mathcal{N}})$ with "diffusive broadening", (*"Mott tracks"* – !).
- Etc.

Our conclusion: Quantum Mechanics and its foundations are well and alive. There are plenty of beautiful new experiments testing fundamental aspects of Quantum Mechanics, and there are plenty of interesting problems for theorists to worry about – good luck!

Thank you!

"Vivre et Survivre" – 47 years later

... depuis fin juillet 1970 je consacre la plus grande partie de mon temps en militant pour le mouvement *Survivre*, fondé en juillet à Montréal. Son but est la lutte pour la survie de l'espèce humaine, et même de la vie tout court, menacée par le déséquilibre écologique croissant causé par une utilisation indiscriminée de la science et de la technologie et par des mécanismes sociaux suicidaires, et menacée également par des conflits militaires liés à la prolifération des appareils militaires et des industries d'armements. ...

Alexandre Grothendieck

Let's take up his struggle again – it is never too late!