Quantum Dynamics of Systems
Under Repeated Observation

Reconstruction of Structure from Unstructured Perception

Jürg Fröhlich (ETH Zurich)

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Outline

We start by presenting a short summary of examples of “effective dynamics” in quantum theory. We then study more closely the effective quantum dynamics of systems interacting with a long chain of independent probes, one after another, which, afterwards, are subject to a projective measurement and are then lost. This leads us to develop a theory of indirect measurements of time-independent quantities (non-demolition measurements). Next, the theory of indirect measurements of time-dependent quantities is outlined, and a new family of diffusion processes – quantum jump processes – is described. Some open problems are proposed.

In memory of my friend the late Claude Itzykson
Credits and Contents

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1. Examples of effective (quantum) dynamics

Here is a list of examples of effective quantum dynamics that are of obvious physical interest and quite non-trivial to analyze:

- **Dynamics in the mean-field regime:** Very high density of particles, very weak two-body interactions; (first studied by Klaus Hepp). The mean-field limit is a classical (field-, or continuum-) limit of QTh, and one can use, e.g., Egorov-type theorems to analyze it. It is the converse of the process of quantizing continuum theories of matter, such as the Vlasov- and the Hartree equations; ↗ “atomism as quantization”. – Other regimes: Gross-Pit. lim, ...

- **Particle limit of continuum theories:** E.g., Hartee solitons as point-like particles exhibiting damped Newtonian motion – possibly interesting in cosmology!

- **Kinetic- or Van Hove regime:** Weak interaction of a “small” system with an infinite (thermal) reservoir; (time rescaled by inverse square of coupling constant) → “Return to Equilibrium”, Approach to a NESS, etc.

  Mathematical methods: Singular perturbation theory, e.g., in the form of the BFS Feshbach-RG
Effective dynamics - ctd.

▶ Isothermal processes: Quasi-static motion of “small” system coupled to a thermostat – isothermal theorem \(\simeq\) adiabatic theorem.

▶ Relaxation to Ground-States & (Quantum) Brownian Motion: “Small” system coupled to \(\infty\)-extended quantized harmonic wave medium at \(T = 0\) relaxes to its ground-state; (F-Gr-Schl, DeR-K). A particle with internal degrees of freedom cpld. to modes of harmonic thermostat (consisting of, e.g., an ideal NR Bose gas) moving in \(\mathbb{Z}^d, d \geq 3\), exhibits diffusive motion \(\rightarrow\) “QBM”!

For highly simplified models, *Einstein relation* betw. particle mobility and diffusion const. can be established; (see DeR-F-Schn). With disorder: Thermal noise always destroys localization; (see F-Sche).

← Expansions around kinetic lim, using (many-scale) cluster exp..

▶ Motion with friction: Particle coupled to a wave medium, such as em field in an optically dense medium, or sound waves in a B-E condensate, emits Cherenkov radiation, causing deceleration of its motion until speed is \(\leq\) speed of wave propagation in medium. Analyzed in mean-field- (FG-Z) and kinetic limit (B-DeR-F). Spectral th.: DeR-F-Pizzo.
Fundamental quantum dynamics of physical systems

Dynamics of systems featuring events – “ETH in QM”:
Fundamental problems concerning *Foundations of Quantum Mechanics* are encountered when one studies the notion of “events” in QM and the question of how events can be recorded, using “instruments” – viz. the theory of *projective measurements*. I have undertaken a considerable effort to elucidate problems surrounding events and projective measurements (of events).\(^1\) My results give rise to the “*ETH approach to quantum mechanics*” – for:

“*Events, Trees, and Histories*”.

⇒ Fundamental qm dynamics of states of phys. systems featuring events can be described in terms of a new kind of *stochastic branching process* whose (non-commutative) state space can be described in terms of families of orthogonal projections. In NR quantum mechanics, branchings are labelled by time and happen continuously. – The fundamental principle underlying the *ETH approach* is the “*Principle of Loss of Access to Information*”.

\(^1\)I’d be happy to talk about my results, but cannot present them here.
2. Systems subject to repeated observation – Haroche-Raimond- and solid-state experiments

The ETH approach represents a "quantum theory without observers". In comparison to the conceptually subtle theory of projective measurements, the theory of indirect (in particular, non-demolition-) measurements is fairly straightforward and can be presented with mathematical precision.

The general Theory of Indirect Measurements of physical quantities – pioneered by Karl Kraus – is the main topic of this lecture.

Karl Kraus (1938-1988)
A metaphor for the theory of indirect observations

Plato’s Allegory of the Cave – ‘Politeia’, in: Plato’s ‘Republic’

As Plato was anticipating, more than 350 years BC, all we “prisoners of our senses” are able to perceive of the world are “shadows of reality”, in the form of long streams of crude, uninteresting, directly perceptible signals (= “projective measurements”) from which well structured, meaningful facts and events can be reconstructed. Socrates explains: philosophers = mathematicians and theoretical physicists are “liberated prisoners” who are able to reconstruct the fabric of reality from the shadows it creates on the wall of the cave.
Systems/experiments to be studied

Sketch of the Haroche-Raimond experiment:

$B$: atom gun, $R_1$: State prep., $C$: Cavity, $R_2$: . . . , $D$: Detector

Sketch of a putative solid-state experiment:

Fig. 4: Experimental setup to study microwave field states with the help of circular Rydberg atoms (see text).
Capture of Sketches

Isolated system $S := E \lor P$, where $P = \text{cavity } C/\text{quantum dot}$, $E = \text{“environment/equipment”}$ consisting of:

(1) Probes: Independent atoms $A_1, A_2, \ldots$ prepared in $R_1/\text{indep.}$ $e^- \text{ traveling through } T-$shaped wires. During time interval $[(m - 1)\tau, m\tau)$, $m^{th}$ atom streams through cavity/$m^{th}$ $e^-$ travels from $e^-$-gun to one of the two detectors $D_L, D_R$; $\tau = \text{duration of a measurement cycle}$.

(2) an atom detector $D/\text{two electron detectors } D_L, D_R$.

It is a little easier to picture how the solid-state experiment works:

Observables referring to quantum dot $P$:

$$\mathcal{O}_P := \{\text{functions of } e^-\text{-number operator } N\}$$

Observables referring to $E$:

$$\mathcal{O}_E = \{1_P \otimes 1_{e_1^-} \otimes \cdots \otimes X_{e_m^-} \otimes 1_{e_{m+1}^-} \otimes \cdots \} m=1,2,3,\ldots,$$
Description of solid-state experiment

where the operator $X_{e_m^-}$ acts on the one-particle Hilbert space of the $m^{th}$ electron traveling through the $T-$ shaped wires towards $D_L, D_R$, resp. It is given by

$$X_{e_m^-} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},$$

with infinitely degenerate eigenvalues $\xi = \pm 1$:

$$\xi = +1 \leftrightarrow e_m^- \text{ hits } D_L, \quad \xi = -1 \leftrightarrow e_m^- \text{ hits } D_R.$$

From now on, “$L$” is replaced by $+1$ and “$R$” by $-1$. The eigen-projection of $X_{e_m^-}$ corresponding to the eigenvalue $\xi$ is denoted by $\pi^m_\xi$; $X_{e_m^-}$ is measured around time $m\tau$.

Let $\rho$ denote the state of $S$. Our aim is to determine the probability of the events that, for $j = 1, 2, \ldots, m$, the $j^{th}$ electron hits the detector $D_{\xi_j}$; $m = 1, 2, 3, \ldots$. 
The LSW formula

For (strictly) independent electrons \(^2\), this probability is given by a formula proposed by Lüders, Schwinger and Wigner (LSW):

\[
\mu_\rho(\xi_1, \xi_2, \ldots, \xi_m) = \text{tr}(\pi_{\xi_m}^m \cdots \pi_{\xi_1}^1 \rho \pi_{\xi_1}^1 \cdots \pi_{\xi_m}^m)
\]  

(1)

Since \(\pi_{\xi_1}^1 + \pi_{\xi_1}^{-1} = 1\), \(\forall m\), and because of cyclicity of the trace,

\[
\sum_{\xi_m} \mu_\rho(\xi_1, \xi_2, \ldots, \xi_{m-1}, \xi_m) = \mu_\rho(\xi_1, \xi_2, \ldots, \xi_{m-1})
\]

Thus, by a lemma due to Kolmogorov, \(\mu_\rho\) extends to a measure on the space, \(\Xi\), of “histories” (= \(\infty\) long measurement protocols \(\xi = (\xi_j)_{j=1}^{\infty}\)), equipped with \(\sigma\)-algebra, \(\Sigma\), generated by cylinder sets.

First, consider the situation where the passage of \(e^-\)'s from the electron gun through the \(T\)-shaped wire to one of the detectors \(D_\xi, \xi = \pm 1\), does not affect the charge, \(\nu\), of the quantum dot \(P\), which is a conserved quantity \(\rightarrow\) “non-demolition measurements”.

\(^2\)the property of strict indep. of \(e^-\)'s is a special case of “decoherence”
Non-demolition measurement & exchangeable probabilities

Then one can argue that the measure $\mu_\rho$ is exchangeable:

$$\mu_\rho(\xi_{\sigma(1)}, \ldots, \xi_{\sigma(m)}) = \mu_\rho(\xi_1, \ldots, \xi_m),$$

for all permutations, $\sigma$, of $\{1, \ldots, m\}$, for arbitrary $m < \infty$. It then follows from De Finetti’s Theorem that

$$\mu_\rho(\xi_1, \ldots, \xi_m) = \int_{\Xi_{\infty}} dP_\rho(\nu) \prod_{j=1}^{m} p(\xi_j | \nu)$$

Here $\Xi_{\infty}$ is the spectrum of the algebra of bounded measurable functions on $\Xi$ that are measurable at $\infty^3$. $\Xi_{\infty}$ is the “space of facts”, or “Dinge an sich”, i.e., the true reality Plato is talking about, whereas the measurement protocols $\xi_m := (\xi_j)_{j=1}^{m}$, $m < \infty$, are the shadows on the wall of the cave that the prisoners are able to perceive, as we shall now explain!

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$^3$equivalence classes w.r. to a measure class (determined by normal states of $S$) of functions on $\Xi$ not depending on any finite nb. of measurement outcomes.
Interpretation of $\Xi_{\infty}$ in the solid-state experiment

Suppose that every electron traveling from the $e^{-}$-gun to one of the detectors $D_{\pm 1}$ is prepared in the same one-particle state $\phi_0$. Assuming that the charge operator, $\mathcal{N}$, of the quantum dot $P$ is a conservation law, the time evolution of the state $\phi_0$ during one measurement cycle is given by $U_\nu \phi_0$, where $U_\nu$ is a unitary operator on the one-electron Hilbert space depending on the charge $\nu$ of $P$: The charge ($\propto$ nb. of $e^{-}$) bound by $P$ creates a “Coulomb blockade” in the right arm of the $T-$ shaped wire; whence the larger $\nu$, the more likely it is that an electron in the wire will be scattered onto the detector $D_1 \equiv D_L$.

The projection of one-electron wave functions that vanish identically near $D_{-\xi}$ is denoted by $\pi_\xi$. The probability, $p(\xi|\nu)$, that an $e^{-}$ hits $D_\xi$ is given by Born's Rule

$$p(\xi|\nu) = \langle \phi_0, U_\nu^* \pi_\xi U_\nu \phi_0 \rangle,$$  \hspace{1cm} (4)

and the space $\Xi_{\infty}$ of “Dinge an sich” is given by

$$\Xi_{\infty} = \text{spec}(\mathcal{N}) = \{0, 1, 2, \ldots, N\}, \quad N < \infty, \quad \mathcal{N} = \text{charge operator of } P.$$
3. Indirect Non-Demolition Measurements: General Results

Let us consider a slightly more general context: \( \mathcal{X}_S \) is the space of possible outcomes of probe measurements, (with \( \mathcal{X}_S = \{-1, +1\} \), in the solid state experiment), and let \( \Xi = \mathcal{X}_S^\times \mathbb{N} \) be the “space of histories”. Assuming “decoherence” for consecutive probe measurements, the measures \( \mu_\rho \) on \( \Xi \) can be decomposed over the spect., \( \Xi_\infty \), of equivalence classes of functions measureable at \( \infty \):

\[
\mu_\rho(\xi) = \int_{\Xi_\infty} \text{d}P_\rho(\nu) \mu(\xi|\nu),
\]

where the measures \( \mu(\cdot|\nu) \) are mutually singular, and \( P_\rho(\Delta) \) is the Born probability of observing a “fact” belonging to the set \( \Delta \in \Xi_\infty \), given that the system \( S \) has been prepared in state \( \rho \). Actually, (assuming “asymptotic abelianess”) the measures \( \mu(\cdot|\nu) \) come from normal states of \( S \), and the “space of facts” \( \Xi_\infty \) can be shown to be contained in or equal to the spectrum of an algebra, \( \mathcal{E}_\infty \), of operators at time \( t = \infty \) in the center of the algebra of “observables” of \( S \); (BFFS).
Basic assumptions

As in the example of the solid-state experiment, we will henceforth assume\(^4\) that:

(i) The measures \(\mu_\rho\) are exchangeable (non-demolition observations), so that

\[
\mu(\xi_m|\nu) = \prod_{j=1}^{m} p(\xi_j|\nu).
\]

(ii) The space of “facts” is a finite set of points (“charge values”) \(\Xi_\infty = \{0, 1, 2, \ldots, N\}\), for some \(N < \infty\). (6)

(iii) We also assume that \(p(\xi|\cdot)\) separates points of \(\Xi_\infty\): There exists \(\kappa > 0\) such that

\[
\min_{\nu_1 \neq \nu_2} |p(\xi|\nu_1) - p(\xi|\nu_2)| \geq \kappa > 0, \quad \text{for some } \xi \in \mathcal{X}_S. \quad (7)
\]

\(^4\)these assumptions can and have been generalized
Summary of main results

Equivalence classes of functions on the space $\Xi$ of histories meas. at $\infty$ form an abelian algebra isomorphic to the algebra of “observables at infinity” (= funs. on the “space of facts” $\Xi_\infty$), which is isomorphic to $\text{Diag}(N+1)$. An example of an “observable at infinity” is the “asymptotic frequency” of an event $\xi \in \mathcal{X}_S$: We define the frequencies

$$f^{(l,l+k)}_\xi(\xi) := \frac{1}{k} \left( \sum_{j=l+1}^{l+k} \delta_{\xi,\xi_j} \right), \quad \text{with} \quad \sum_\xi f^{(l,l+k)}_\xi(\xi) = 1. \quad (8)$$

**Main results:**

(1) **Law of Large Numbers for exchangeable measures:** The asymptotic frequency satisfies

$$\lim_{k \to \infty} f^{(l,l+k)}_\xi(\xi) =: p(\xi | \nu), \quad (9)$$

for some point (or “fact”) $\nu \in \Xi_\infty$. (Special case: Experiments explained at the beginning.)
With each $\nu \in \Xi_\infty$ we associate a subset

$$\Xi_\nu(l, k; \varepsilon) := \{ \xi \mid |f_{\xi}^{(l, l+k)}(\xi) - p(\xi|\nu)| < \epsilon_k \},$$

where

$$\epsilon_k \rightarrow 0, \sqrt{k} \epsilon_k \rightarrow \infty, \quad \text{as } k \rightarrow \infty$$

(2) **Distinguishability:** It follows from Hyp. (7) and definition (8) that, for $k$ so large that $\epsilon_k < \kappa/2$,

$$\Xi_{\nu_1}(l, k; \varepsilon) \cap \Xi_{\nu_2}(l, k; \varepsilon) = \emptyset, \quad \nu_1 \neq \nu_2.$$

(3) **Central Limit Theorem:** $\Rightarrow$ Under suitable hypotheses on the states $\rho$, e.g., (i) through (iii),

$$\mu_\rho \left( \bigcup_{\nu} \Xi_\nu(l, k; \varepsilon) \right) \rightarrow 1, \quad \text{as } k \rightarrow \infty.$$

(1), (2) & (3) $\Rightarrow$ As $m \rightarrow \infty$, every history $\xi_m$ of measurement outcomes determines a unique point ("charge") $\nu \in \Xi_\infty$; (error $\rightarrow 0$, as $m \rightarrow \infty$).
hypothesis testing – ctd.

Moreover, *Born’s Rule* holds: \( \mu_\rho(\Xi_\nu(l, k; \xi)) \to P_\rho(\nu), \ k \to \infty. \)

(4) **Theorem of Boltzmann-Sanov** \( \Rightarrow \) If the measures \( \mu_\rho \) are exchangeable one has that

\[
\mu(\Xi_{\nu_1}(l, k; \xi)|\nu_2) \leq C e^{-k\sigma(\nu_1||\nu_2)}
\]

where \( \sigma \) is the relative entropy of the distribution \( p(\cdot|\nu_1) \) given \( p(\cdot|\nu_2) \).

(5) **Theorem of Maassen and Kümmerer** \( \cdots \Rightarrow \) In the Haroche-Raimond experiment described above, the state of \( S \), restricted to \( B(\mathcal{H}_\rho) \), approaches a state, \( \rho^\nu \), with a fixed number, \( \nu \), of photons in the cavity \( P(\equiv C) \), as \( k \to \infty \): “Purification”!

(Analogous results for solid-state experiment.)

The theory of indirect measurements outlined here only concerns measurements of time-independent “facts”, which correspond to points in \( \Xi_\infty \) (non-demolition measurements!). However, most interesting facts depend on time, i.e., are “events”, and \( \Xi_\infty = \emptyset \)! Thus, we must ask how one can acquire information concerning events indirectly, through repeated direct measurements of quantities corresp. to operators in \( \mathcal{O}_E \).

We consider an isolated physical system $S = P \lor E$, as before.
States of $S$ are given by density matrices, $\rho_S$, acting on a Hilbert space $\mathcal{H}_S = \mathcal{H}_P \otimes \mathcal{H}_E$, where $\mathcal{H}_P = \mathbb{C}^{N+1}$, for some $N < \infty$. When restricted to observables of $P$, states are given by density matrices

$$\rho_P := \text{tr}_E \rho_S.$$  \hspace{1cm} (11)

Hilbert space of a single probe $A_j$: $\mathcal{H}_{A_j} \simeq \mathcal{H}_A$
Initial state of each probe $A_j$: $\phi_0 \in \mathcal{H}_A$.
Reference state in $\mathcal{H}_E$: $\bigotimes_{j=1}^{\infty} \phi^{(j)}_0$, $\phi^{(j)}_0 = \phi_0, \forall j$.
Space $\mathcal{H}_E =$ completion of linear span of vectors $\bigotimes_{j=1}^{\infty} \psi^{(j)}$, with $\psi^{(j)} = \phi_0$, except for finitely many $j$.
For each probe $A_j$, the same observable, represented by the operator

$$X = \sum_{\xi \in \mathcal{X}_S} \xi \pi_\xi,$$  \hspace{1cm} (12)

acting on $\mathcal{H}_{A_j}$, is measured in a detector $D$. 
The formalism

(D will not play any role in the following, hence is omitted.) During the $j^{th}$ measurement cycle $(t_{j-1}, t_j]$, only $A_j$ interacts with $P$, at time $t_j$. Measurement results for probes $A_1, \ldots, A_{j-1}: \xi_{j-1} = (\xi_k)_{k=1}^{j-1}$.

Some notations:

- $\rho_P^{(j-1)} \equiv \rho^{(j-1)}(t_{j-1}, \xi_{j-1})$, $t_{j-1} := (t_k)_{k=1}^{j-1}$: State of $P$ right after interaction with $A_{j-1}$, at time $t_{j-1}$.
- Let $\mathcal{N}$ be a “charge operator” acting on $\mathcal{H}_P$ with simple spec$(\mathcal{N}) = \{0, 1, \ldots, N\}$, $N < \infty$: $E_\nu$: spectral projection of $\mathcal{N}$ corresp. to ev $\nu$.

Time evolution of $P \lor A_j$ from time $t_{j-1}$ to time $t_j$ right before $A_j$ is subject to projective measurement of $X$ with outcome $\xi_j$, is given by:

$$\rho_{P \lor A_j} := \sum_{\nu, \nu'} E_\nu e^{-i(t_j-t_{j-1})H_P} \rho_P^{(j-1)} e^{i(t_j-t_{j-1})H_P} E_{\nu'} \otimes U_\nu |\phi_0\rangle \langle \phi_0| U_{\nu'}^*, \quad (13)$$

where $H_P$ is the Hamiltonian of $P$, and $U_\nu$ is a unitary on $\mathcal{H}_A$ mapping the initial state, $\phi_0$, of $A_j$ onto the state of $A_j$ right after its interaction with $P$, given that the charge of $P$ at time $t_j$ is given by $\nu$. Then the observable $X$ is measured projectively for $A_j$, with result $\xi_j \in \mathcal{X}_S$. 
Formalism – ctd.

This yields a recursion formula for the state $\rho^{(j)}_P$:

$$
\rho^{(j)}_P = Z_{\xi_j}^{-1} V_{\xi_j} e^{-i(t_j-t_{j-1})H_P} \rho^{(j-1)}_P e^{i(t_j-t_{j-1})H_P} V_{\xi_j}, \quad (14)
$$

where $Z_{\xi}$ is a normalization factor, and $V_{\xi}$ is given by

$$
V_{\xi} = \sum_{\nu} V_{\xi}(\nu), \text{ where } V_{\xi}(\nu) := E_{\nu} \sqrt{p(\xi|\nu)}
$$

with $p(\xi|\nu) := \langle U_{\nu}\phi_0, \Pi_{\xi} U_{\nu}\phi_0 \rangle$; (↗ (iii), Sect. 3). Note that

$$
V_{\xi} = V_{\xi}^*, \ [V_{\xi}, N] = 0, \forall \xi, \text{ and } \sum_{\xi' \in \mathcal{X}} V_{\xi'}^2 = 1. \quad (15)
$$

The recursion formula (14) yields a trajectory of states of the subsystem $P$ (the cavity/quantum dot) given by

$$
\rho_t(t_j, \xi_j) := e^{-i(t-t_j)H_P} \rho^{(j)}_P(t_j, \xi_j) e^{i(t-t_j)H_P}, \ t_j < t < t_{j+1}. \quad (16)
$$
Averaged time-evolution of state of $P$

We now suppose that the times $t_j$ of interaction between the probes $A_j$ and the subsystem $P$ are Poisson distributed, with rate $\gamma = 1, \forall j$. Fixing a time $t$ and taking an average over measurement times and measurement outcomes, we find that

$$\mathbb{E}[\rho_t(t, \xi, \xi')] = e^{t \mathcal{L}} \rho,$$  \hspace{1cm} \text{(17)}

where $\rho$ is the initial state of the subsystem $P$ (at time $t = 0$), and $\mathcal{L}$ is a Lindblad generator given by

$$\mathcal{L} \rho = -i \text{ad}_{H_P} (\rho) + \left( \sum_{\xi \in \mathcal{X}_S} V_\xi \rho V_\xi \right) - \rho.$$  \hspace{1cm} \text{(18)}

Eq. (16) is called “unravelling” of the Lindblad evolution (17); it appears as the integrand in the Dyson expansion of the right side of (17), with the second term on the right side of (18) treated as the perturbation.
Main result

We suppose that the “Basic Assumptions”, (i)-(iii), of Sect. 3 are valid. We assume furthermore that

\[ H_P = \varepsilon h_p, \quad \text{for some } \varepsilon > 0, \]  

(19)

and we rescale time: \( t = \varepsilon^{-2} \tau \). We define a continuous-time Markov jump process, with state space \( \text{spec}(\mathcal{N}) \), paths \( \nu_\tau(\omega), \omega = (t, \xi) \), and transition function generated by the Markov kernel:

\[
Q(\nu, \nu') = -\frac{|\langle \nu | h_P | \nu' \rangle|^2}{\sum_{\xi \in \mathcal{X}} V_\xi(\nu)V_\xi(\nu') - 1} + cc, \quad \nu \neq \nu',
\]

with \( Q(\nu, \nu) = \cdots \geq 0, \forall \nu \).

We are now prepared to state our main result, (which is a “baby version” of the “ETH approach” to QM!).
Main result – ctd.

Theorem.

Convergence of qm evolution to Markov jump process:

$$\lim_{\varepsilon \searrow 0} \mathbb{E}\left[ \rho_{\varepsilon^{-2\tau}}(\omega = (t, \xi)) \right] = e^{-\tau Q} \rho_0,$$

where $\rho_0 = \text{Diag} \left( \langle \nu | \rho | \nu \rangle \right)$;

The state $\rho_{\varepsilon^{-2\tau}}(\omega = (t, \xi))$ approaches in law a diagonal matrix, $\text{Diag}(\delta_{\nu, \nu_t(\omega)})$.

Numerical simulation for the behaviour of the diagonal matrix elements of $\rho_{\varepsilon^{-2\tau}}(t, \xi)$ in the special case where $N = 1$ (i.e., $\mathcal{H}_P = \mathbb{C}^2$), for small $\varepsilon$:
5. Open Problems, Conclusions

- More general models of probes and “cavities”; in particular:
- Weakly correlated probes; infinite-dimensional state spaces for cavity, $P$; operators $\mathcal{N}$ with continuous spectrum (!); . . .
- More general models of indirect measurements of time-dependent quantities.

  *Important Example – to be carried out more fully:* Consider observables, $\mathcal{N}$, with continuous spectrum, $\sigma(\mathcal{N}) \simeq \mathbb{R}^d$. Then $H_P$ may generate dynamics describing inertial motion on $\sigma(\mathcal{N})$, and the full dynamics of $P$ then describes tracks on $\sigma(\mathcal{N})$ with “diffusive broadening”, (“Mott tracks” – !).

- Etc.

**Our conclusion:** Quantum Mechanics and its foundations are well and alive. There are plenty of beautiful new experiments testing fundamental aspects of Quantum Mechanics, and there are plenty of interesting problems for theorists to worry about – good luck!

*Thank you!*
“Vivre et Survivre” – 47 years later

... depuis fin juillet 1970 je consacre la plus grande partie de mon temps en militant pour le mouvement Survivre, fondé en juillet à Montréal. Son but est la lutte pour la survie de l’espèce humaine, et même de la vie tout court, menacée par le déséquilibre écologique croissant causé par une utilisation indiscriminée de la science et de la technologie et par des mécanismes sociaux suicidaires, et menacée également par des conflits militaires liés à la prolifération des appareils militaires et des industries d’armements. ...

Alexandre Grothendieck

Let’s take up his struggle again – it is never too late!