Some new results for the Ising model on the Cayley tree

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joint work with

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Preamble

What these results are about

- New Gibbs sates
- found by Rozikov and Ramatullaev, rather recently '08
- called weakly peridioc
- not "Translation Invariant"
- not "Dobrushin like" states

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Outline

Reminder (and two methods)

- The Ising model on the Cayley tree τ_k
- The Tanslation Invariant states
- The B-G (Blekher-Ganikhodzhaev) states
- The method of Recursive equations
- Contour method
- Other "Dobrushin-like" states

Weakly periodic Gibbs states

- Alternative construction of R-R (Rozikov-Ramatullaev) states
  How to show extremality
- A construction of more general weakly periodic Gibbs states

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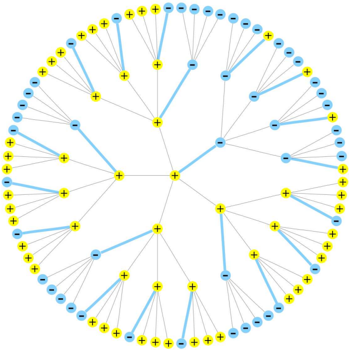

Each vertex has $k+1$ neighbors (here $k=4$)

$\sigma_x = \pm 1$ Energy : $H(\boldsymbol{\sigma}) = -J \sum_{\langle x,y \rangle} \sigma_x \sigma_y$

Gibbs measures:

$\mu_n(\boldsymbol{\sigma}) = Z_n^{-1} \exp[(1/T) \sum_{\langle x,y \rangle \in V_n} J \sigma_x \sigma_y + \sum_{x \in W_n} h_x \sigma_x]$

$h_x \sim \sum_{y \in S_x} \sigma'_y$ (b.c.) or given set of reel numbers (g.b.c.)

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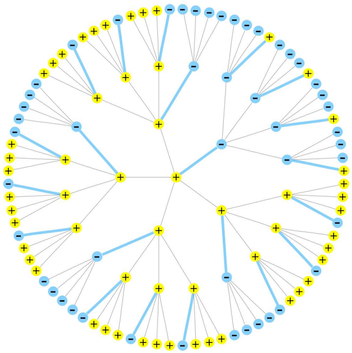

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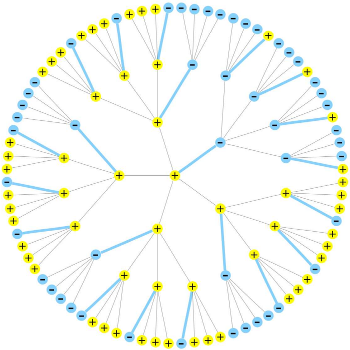

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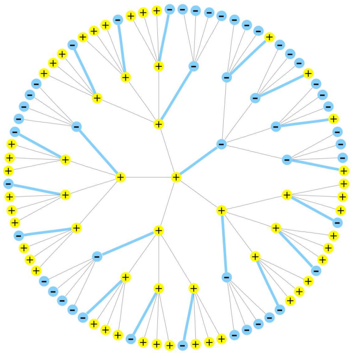

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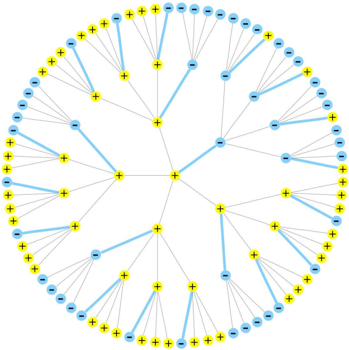

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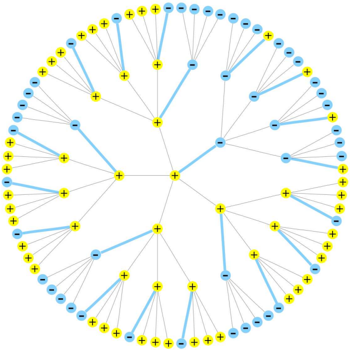

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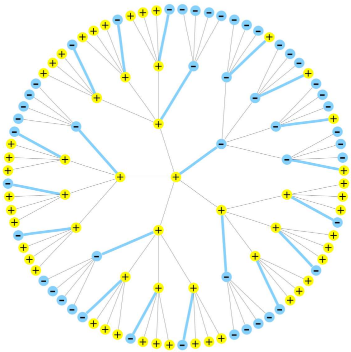

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Translation Invariant Sates

- For $T < T_c$: the states μ^+ and μ^- corresp. to $\sigma' = +1$ and $\sigma' = -1$ are extremal
- For $T \geq T_c$: the states μ^0 and corresp. to $h = 0$ (free b.c.) is extreme (and unique)

$$T_c = J/\arctan(1/k)$$

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- For $T < T_c$: there exists $\mu^{\pm}$ constructed with mixtures of $+$ and $-$ b.c.
- They differ from the μ^{+} and μ^{-} states
- One can construct uncountable many such (extreme) sates

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Recursive equations method

• measures μ_n and μ_{n-1} are said compatible if

$$\sum_{\sigma_x, x \in W_n} \mu_n(\sigma_n) = \mu_{n-1}(\sigma_{n-1})$$

• This holds iff the h satisfy

$$h_x = \sum_{y \in S(x)} f(h_y)$$

• In case one is interested in TI states, one has to look at solutions of

$$h = k\ f(h)$$

• Depending on T and J, there exist either 3 sol. or only 1 sol. and the value of T_c is obtained by solving $k\ f'(h) = 1$

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Contours method

- Consider, in a finite domain, a configuration σ that disagree with a ground state configuration say $+$
$+$ sites are called correct and $-$ sites are calle incorrectotherwise.
The set of incorrect sites decomposes into connected components called contours.

- Energy estimates:
the cost is proportional to the boundary of the contour.
The boundary is of the size of the interior

$$e^{-(J/T)\,\mathrm{Cte}\,|\Gamma|}$$

- Entropy estimates:
The number of connected subgraphs with n bonds, containing a given vertex, is bounded from above by

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Others "Dobrushin-like" states

- The measure μ^0 is extreme iff $T \geq T_{SG} \geq J/\arctan(1/\sqrt{k})$ (Bleher-R.-Zagrebnov '95), (Ioffe '96)

$\Rightarrow$ For $T_{SG} \leq T < T_c$, the $+$, $-$ and free states are extreme.

- In this range one can construct extreme states mixtures of $+$ (or $-$) and free boundary conditions (Akin-Rozikov-Temir '11)

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Weakly Periodic sates

• Such states have been constructed by Rozikv and Ramatullaev in '08. The are neither T.I. nor B-G type. • $k \geq 4$, Recursive equation method, extremality not proven.

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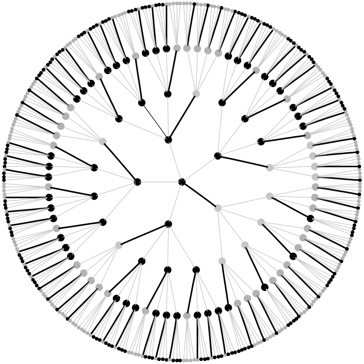

Start with a dimer configuration D of black bonds

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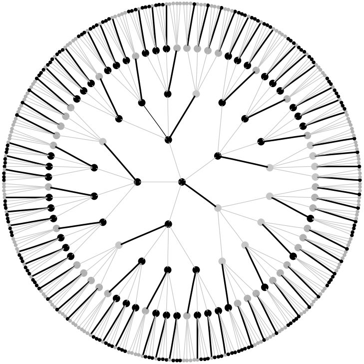
Alternative construction of R-R states



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How to proove extremality

- By contours method
- Sites of a given configuration are correct if they agree with σ^D and incorrect othewise. Then proceed as before to define contours
- Energy estimate for R-R states (proof by induction)

$$H(\sigma_\Gamma) - H(\sigma^D) \geq 2J\,[\,k-3]\,|\mathrm{Int}\,(\Gamma)\,|$$

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A construction of more general weakly periodic Gibbs states

Let $D \subset E$ be an arbitrary collection (finite or infinite) of edges of $\mathcal{T}^k$. As before

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Results

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The configurations σ^{D+} and σ^{D-} are ground state configurations.

Theorem (2)

There exists a value T_k of the temperature, such that for all temperatures $T < T_k$ and all collections D satisfying $(^*)$ the states μ^{D_+} and μ^{D-} are extremal Gibbs states.
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Theorem (2)

There exists a value T_k of the temperature, such that for all temperatures $T < T_k$ and all collections D satisfying $(^*)$ the states μ^{D_+} and μ^{D-} are extremal Gibbs states.
(Here the temperature T_k does not depend on D).

Two remarks

• $D = \varnothing$. In that case the states $\mu^{\varnothing+}$ and $\mu^{\varnothing-}$ are just the $(+)$ and $(-)$ states.

• In the case when D **consists of** a single bond b, the states μ^{b+} and μ^{b-} are among the non-translation invariant states $\mu_S^{\pm}$ constructed by Blekher and Ganikhodzhaev

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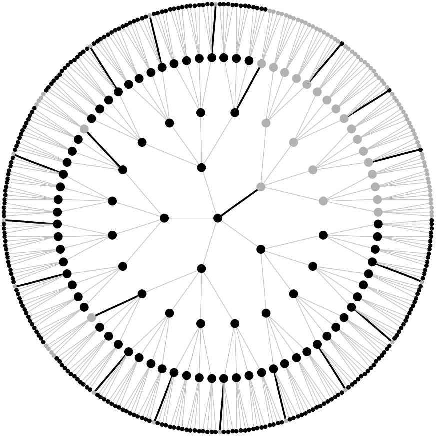
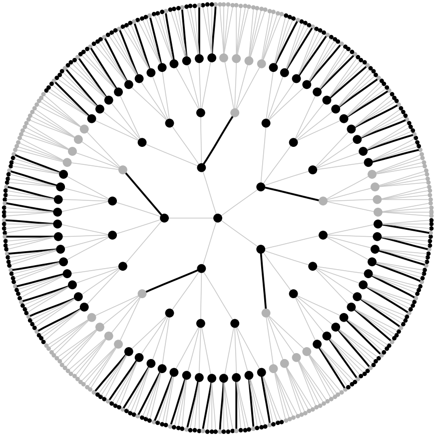
Examples



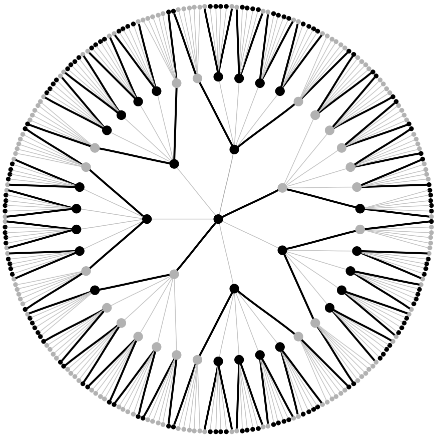
Second dimer covering

Examples



Monomer-dimer covering

Examples



Path-dimer covering

STOP NOW

Thank–You