Papangelou processes and inductive probabilities

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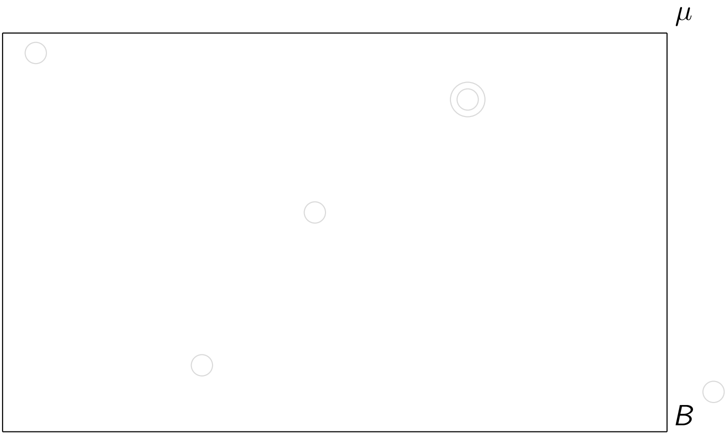
joint work with Hans Zessin

Yerevan, Sept. 4, 2012

The setting

Describing random point configurations



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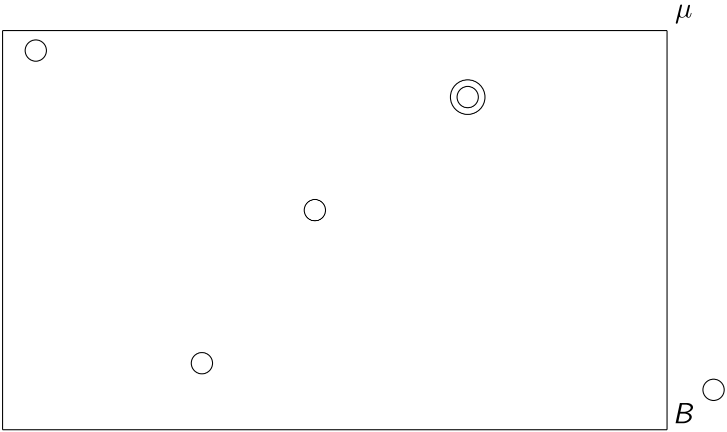

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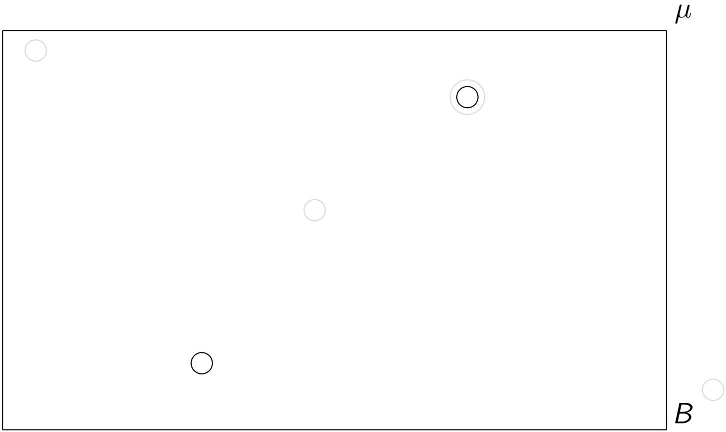

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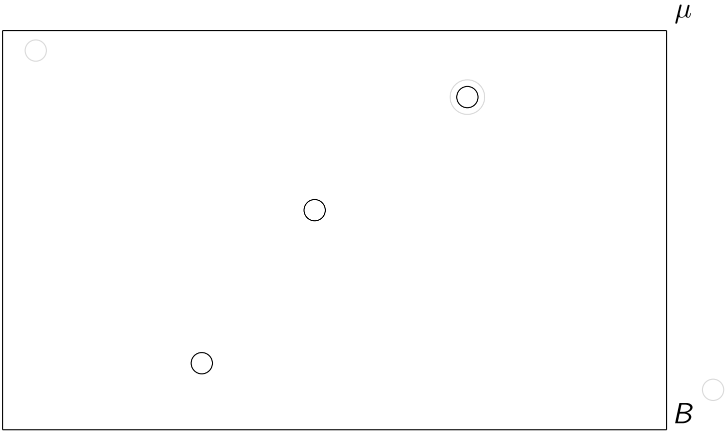

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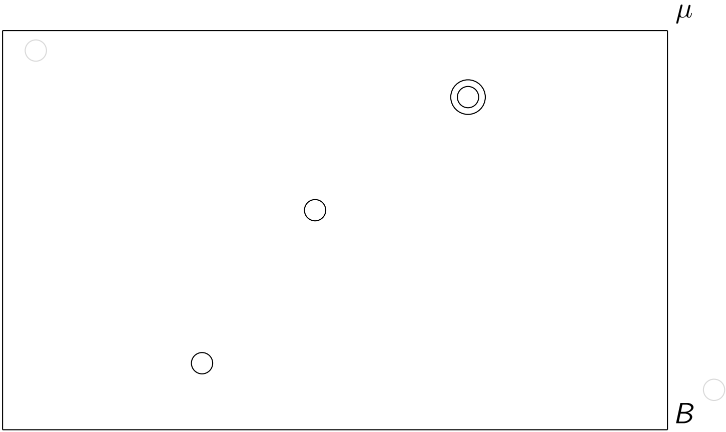

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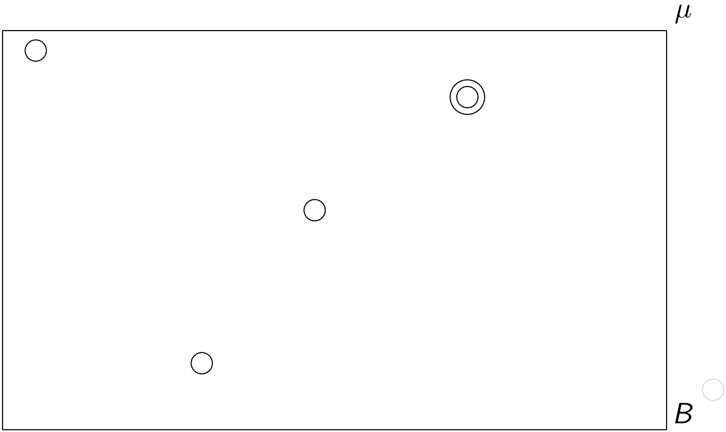

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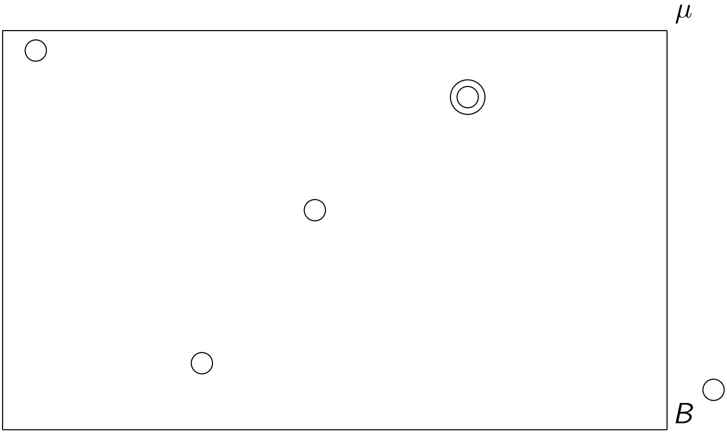

- realize points one by one
- inductive rule $\pi : \mu \mapsto \pi_\mu$
    - symmetry condition
    - integrability condition
    - $\pi_\mu(B)$ expected number of points in B given μ

Question

What is the structure of π under one additional assumption on measurability or stability under certain mappings?

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- observation $X_1, \dots, X_N$ with permutation invariant joint law
- infer on law of X_{N+1} given $X_1, \dots, X_N$
- W. Johnson ($\sim$ 1924), R. Carnap ($\sim$ 1950)

$$(\mathcal{J}) \qquad \mathbb{P}(X_{N+1} = j | X_1, \dots, X_N) = f_j(n_j)$$

- Böge ($\sim$ 1970)

$$(\mathcal{B}) \qquad \begin{aligned} &\mathbb{P}\big(\varphi(X_{N+1}) = i | X_1, \dots, X_N\big) \\ &\quad = \mathbb{P}\big(\varphi(X_{N+1}) = i | \varphi(X_1), \dots, \varphi(X_N)\big) \end{aligned}$$

Postulates are eqivalent to

$$\mathbb{P}(X_{N+1} = j | X_1, \dots, X_N) = \frac{a_j + b n_j}{A + bN}$$

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assume π is a kernel such that for $\mu \mapsto \pi_\mu$,

$(\mathcal{N})$ $\quad 1_{\{y\}^c}\pi_{\mu+\delta_y} = 1_{\{y\}^c}\pi_\mu$

$(\mathcal{S})$ $\quad$ space contains at least 3 elements

Linear Reinforcement (Zessin, R. 12)

Given $(\mathcal{N})$ and $(\mathcal{S})$,

$$\pi_\mu(\mathrm{d}x) = \rho(\mathrm{d}x) + c(x)\mu(\mathrm{d}x) \qquad (1)$$

Sufficiency for singletons (Zessin, R. 12)

(1) is equivalent to

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Existence (Zessin 09; Nehring, Zessin 11)

If $c(x) \in [0,1)$ or if $c(x) < 0$ and $-\frac{\rho(\{x\})}{c(x)} \in \mathbb{N}$, then there exists a unique point process P such that

$$C_{\mathsf{P}}(h) = \iint h(x, \mu + \delta_x)\pi_\mu(\mathrm{d}x)\mathsf{P}(\mathrm{d}\mu). \tag{2}$$

Assume $G : X \to Y$ is a state space transformation s.th. $G\mathsf{P}$ is a point process

State space transformation

If $G\mu_1 = G\mu_2$ implies $\pi_{\mu_1} \circ G = \pi_{\mu_2} \circ G$ for an admissable G, then $G\mathsf{P}$ satisfies (2) with π_μ replaced by $\pi'_\nu = \pi_\mu \circ G$ for μ such that $G\mu = \nu$.

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Question

What happens if π is stable under a huge class of admissable state space transformations?

Stability and sufficiency (Zessin, R. 12)

Assume that there exists P for kernel π and $(\mathcal{S})$. Then the following statements are equivalent

1. $\pi_\mu(\mathrm{d}x) = \rho(\mathrm{d}x) + c\mu(\mathrm{d}x)$ for some $c < 1$,
2. $(\mathcal{N})$ and $x \mapsto \pi_{\delta_x}(\{x\}) - \pi_0(\{x\})$ is constant,
3. $\pi(B)$ is $\sigma(N_B)$-measurable for all closed B,
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1. two versions of the sufficiency postulate yield basically same structure, but differences in detail
2. Examples (Bach, Zessin)
   - MB statistics: $\pi_\mu(\mathrm{d}x) = c(x)\rho(\mathrm{d}x)$
   - BE/FD statisitcs: $\pi_\mu(\mathrm{d}x) = c(x)\big[\rho \pm \mu\big](\mathrm{d}x)$
3. add interactions: $1_{\{y\}^c}\pi_{\mu+\delta_y} = f(\,\cdot\,, y)1_{\{y\}^c}\pi_\mu$

$$\pi_\mu(\mathrm{d}x) = V_\mu(x)\big[\rho(\mathrm{d}x) + c(x)\mu(\mathrm{d}x)\big]$$

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