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Preface

Mathematical physics is dedicated to making rigorous the interplay between mathematics and physics. A number of intriguing problems in analysis are motivated by questions arising from physically interesting phenomena. The goal of the *Entropy and the Quantum* schools has been to introduce young researchers to some of the exciting current topics that involve the analytic setting and which is best understood with a dose of physical intuition.

After the success of the first school in 2009, the organizers found themselves under friendly, yet firm and unescapable, pressure to organize the *Arizona School of Analysis with Applications*, which took place on the campus of the University of Arizona in March 15-19, 2010. Four beautiful lectures were delivered by Rafael Benguria (*Isoperimetric Inequalities for Eigenvalues of the Laplacian*), Laszlo Erdös (*Universality of Wigner Random Matrices*), Michael Loss (*Kinetic Theory and Kacs Master Equation*), and Günter Stolz (*Localization in Disordered Media*). In addition, there were talks by "senior" participants (Alain Joye, Takuya Mine, Luc Rey-Bellet, Shannon Starr, Simone Warzel), and many short talks by junior participants. The range of the subjects and the enthusiasm of the young speakers is testimony of the great vitality of the field.

The lecture notes in this volume reflect the diversity of interests of the participants of the school. They cover topics such as kinetic theory (the article by Carlen, Carvalho, and Loss), isoperimetric inequalities (articles by Benguria and by Gamara, Hasnaoui, and Hermi), Anderson localization for random quantum systems (articles by Stolz and by Joye), quasi-periodic potential (the article of Krüger is directly motivated by the school), random matrix theory (by Maltsev and Schlein), correlations for random analytic functions (by Starr), locality estimates and probabilistic representations of quantum spin systems (articles by Nachtergaele, Vershynina, and Zabrebnov, and by Goldschmidt, Ueltschi, and Windridge).

As usual, the campus of the University of Arizona was sunny and pleasant. Post-lecture discussions took place around the campus, and they were certainly as invigorating as the previous year. The visit to Sabino Canyon, North-East of Tuscon at the foot of the Catalina mountain range, gave the participants some flavor of the beautiful desert area around the town.

The organizers are grateful to the many people who helped us make this school possible. David Gonzalez and Jennifer Hardy were incredibly dedicated and impressively efficient as coordinators of the school. The week ran smoothly thanks to them. We would also like to thank Bruno Nachtergaele for his fidelity and encouragements. This volume of Contemporary Mathematics bears the very nice title that Bill Faris suggested last year. Support for the school was provided mainly by the National Science Foundation (grant DMS-1001153), but we are also happy

to acknowledge the matching support generously donated by the Department of Mathematics at the University of Arizona. Finally, our gratitude goes to all the lecturers, participants, and the authors of the articles in the present volume, for their contribution to the success of the event.

Tucson and Coventry, May 14th, 2011

Robert Sims, Daniel Ueltschi

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