Fluctuations of Last Passage Percolation

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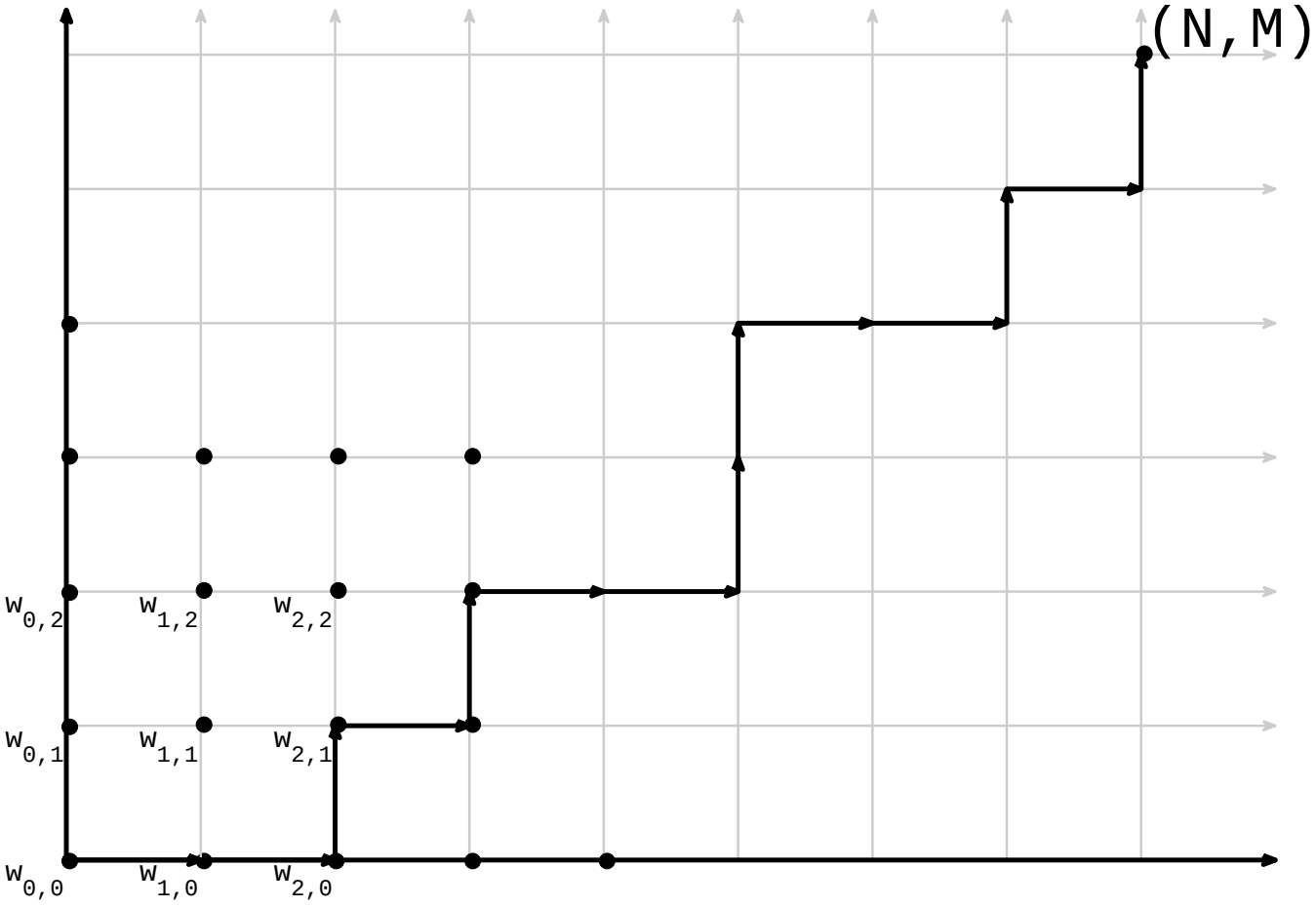
Directed LPP:

- $w_{i,j}$ independent weights
- π directed $\uparrow$, $\rightarrow$ path
- $T(\pi) = \Sigma_{(i,j)\in\pi} w_{i,j}$
- $L(N,M) = \max_{\pi:(0,0)\rightarrow(N,M)} T(\pi)$.

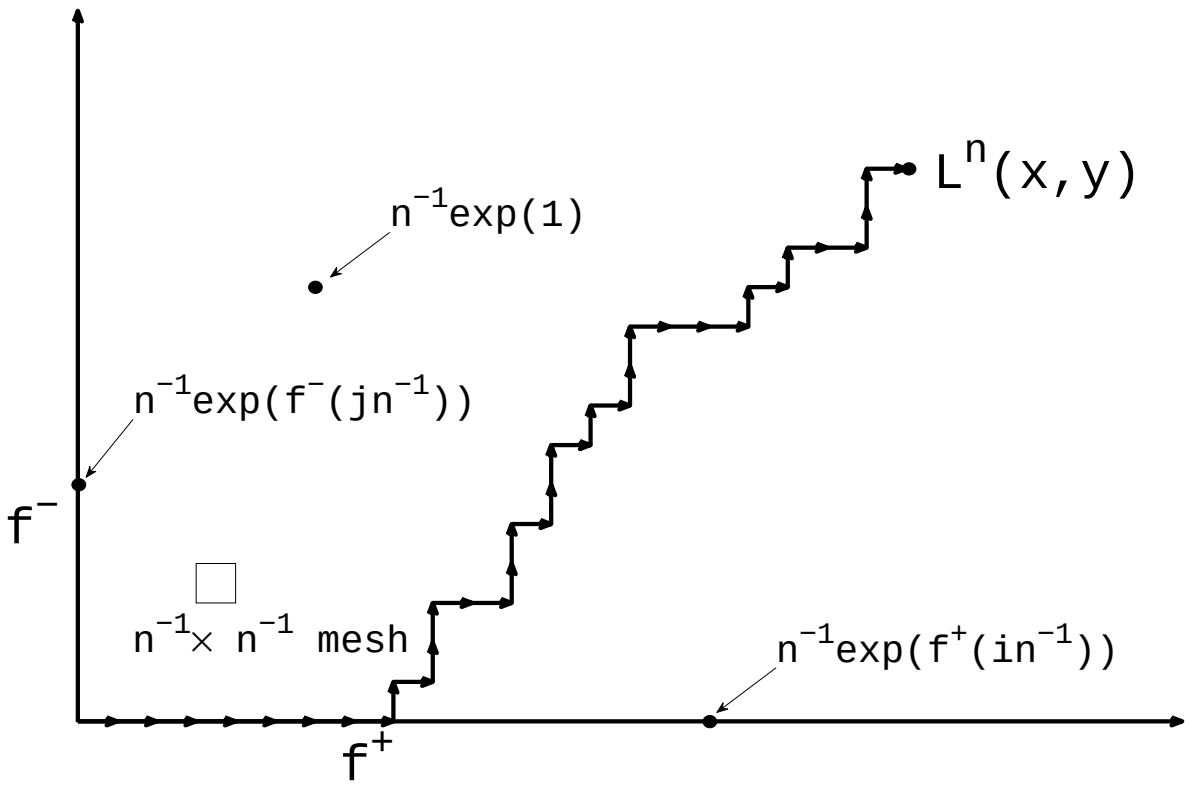


$f^+, f^- : \mathbb{R}^+ \to \mathbb{R}^+$ (nice)

Define LPP^n on $\frac{1}{n}\mathbb{Z} \times \frac{1}{n}\mathbb{Z}$ via weights:

$$w_{\frac{i}{n},\frac{j}{n}} \sim \begin{cases} n^{-1}\exp(1), & i,j \geq 1 \\ n^{-1}\exp(f^+(\frac{i}{n})) & i \geq 1, j = 0 \\ n^{-1}\exp(f^-(\frac{j}{n})) & i = 0, j \geq 1. \end{cases}$$

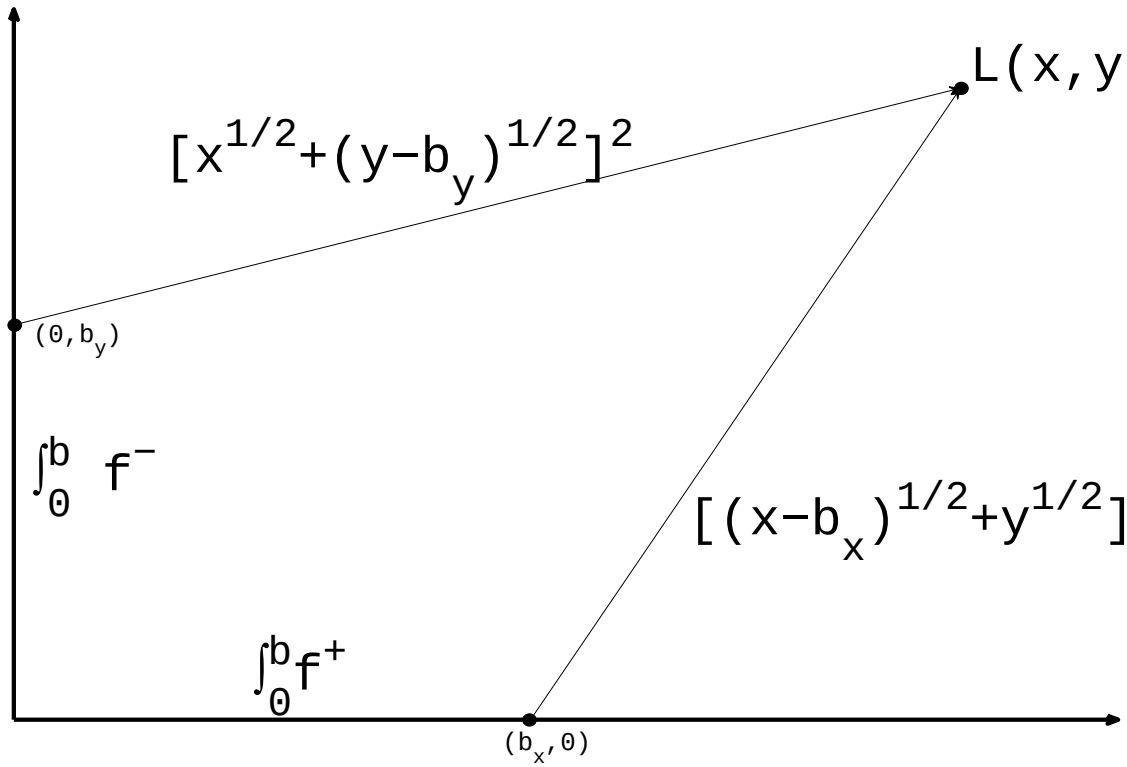
Study $L^n(x,y)$ as $n \to \infty$



Theorem(I.C.) *hydrodynamic theory*: $L^n(x,y) \to L(x,y)$ a.s.; for any $\psi \in C_c^\infty$, $(L^n, \psi)_n \to (L, \psi)$ in probability, where

$$L(x,y) = \sup\left[\int_0^b f(x) + \int_b^{(x,y)} (1+\sqrt{\gamma'})^2\right]$$
$$= \sup\left[\int_0^b f(x) + (\sqrt{x-b_x} + \sqrt{y-b_y})^2\right].$$

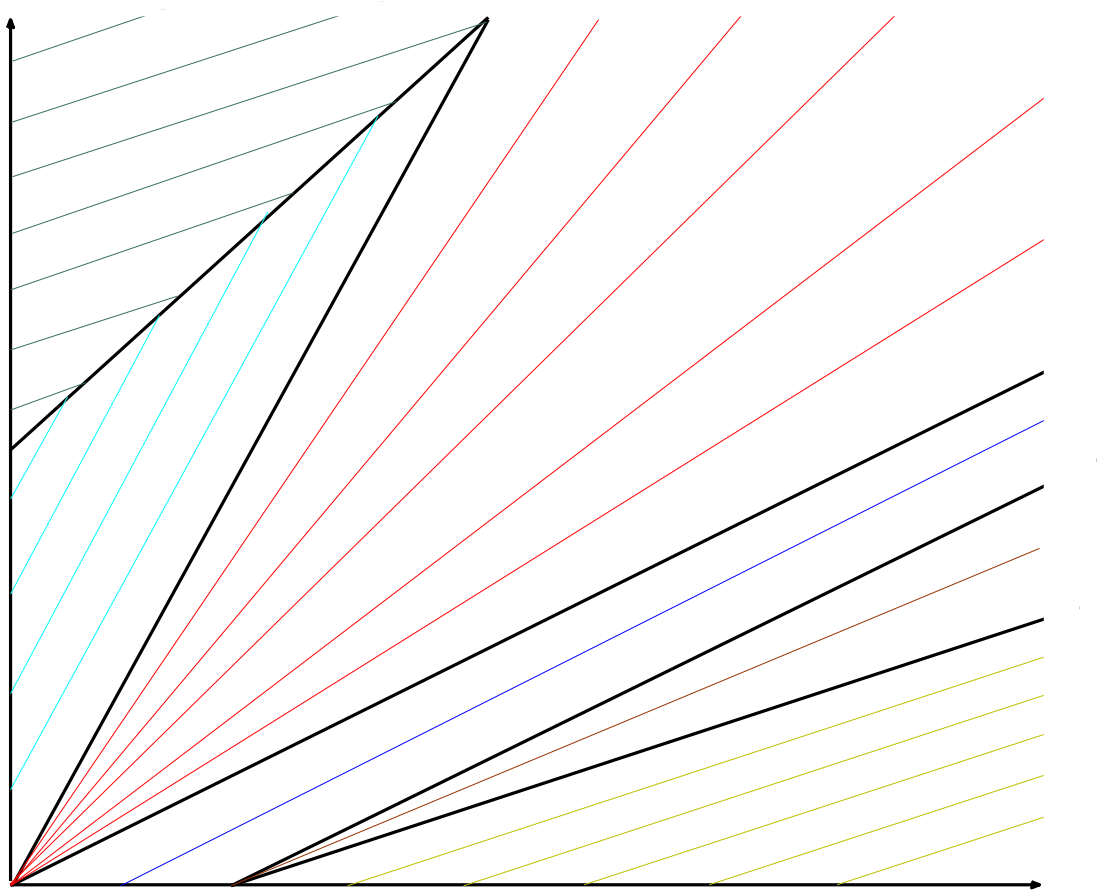
- b on corner boundary. γ straight lines



- Hopf-Lax-Oleinik formula for PDE:

$$\begin{cases} L_x L_y = L_x + L_y & x, y > 0 \\ L(x, y) = f^+(x) & y = 0 \\ L(x, y) = f^-(y) & x = 0. \end{cases}$$

Characteristics point to maximizing pts.

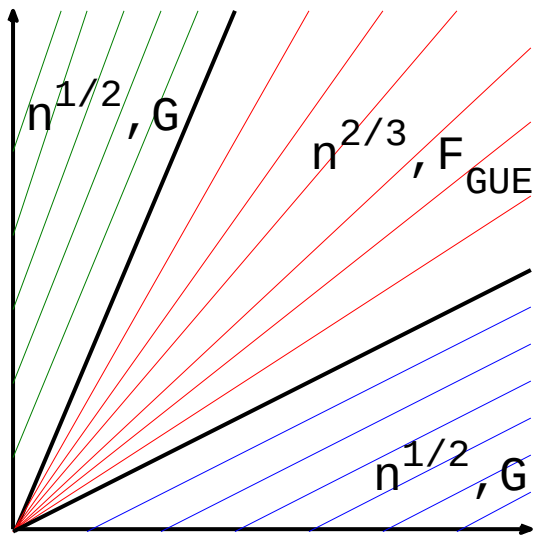


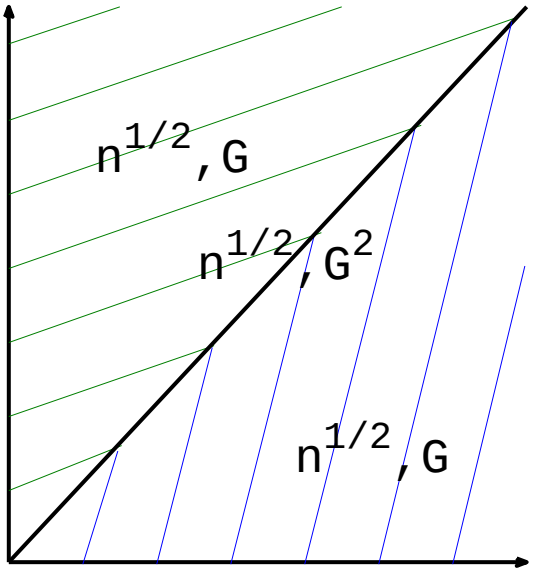
Theorem(I.C.) *fluctuation theory*:

$\mathcal{F}^n(x,y) = [L^n(x,y) - L(x,y)]n^\alpha \Rightarrow D$

where α and D depend on (x,y).

- Char. from origin: $\alpha = 2/3$, D random matrix theory type $(F_{GUE}, F^2_{GOE}, F_0)$
- One bdry char.: $\alpha = 1/2$, Gaussian
- Many bdry char.: $\alpha = 1/2$, maximum of correlated Gaussians





Theorem(I.C.) *fluctuation process*:
Limit process (f.d.d.) for $\{\mathcal{F}^n(x_i, y_i)\}_{i=1}^m$.

• If $x_i, y_i's$ in Gaussian region, limit process is related to Brownian motion.

• If $x_i, y_i's$ in RMT region, in order to get non-trivial correlation must take

$$(x_i, y_i) = (x, y) + n^{-1/3}(\tilde{x}_i, \tilde{y}_i)$$

Converge to Airy_2 process (or $\text{Airy}_{2\to BM}$ or Airy_{stat}).

• Boundary char. = normal CLT

• Origin char. = KPZ CLT.

Previous work:

• $f^+ = f^- = 0$: Rost 81' LLN, Johansson 99' CLT.

• $f^+ = c^+$, $f^- = c^-$: Seppäläinen 97' LLN, Prähofer-Spohn Conjectured CLT 02', Baik-Ben Arous-Péché $c_+ = 0$ CLT, Ferrari-Spohn 05', Ben Arous-Corwin 09'.

• Multipoint CLT: Borodin-Ferrari, Baik-Ferrari-Péché, Corwin-Ferrari-Péché.

Thanks.