The Anderson model on a strip and random matrix theory

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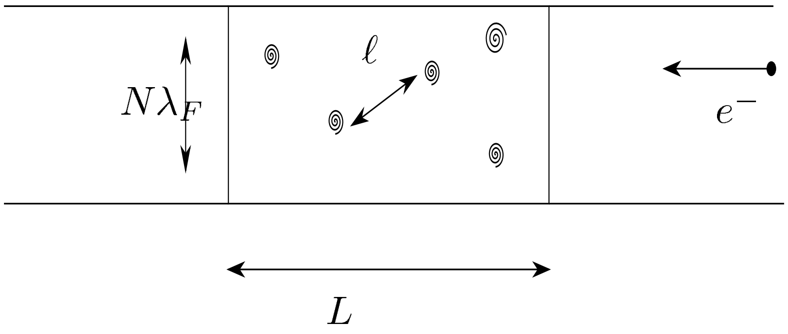
joint work with W. de Roeck

Disordered wires: Phenomenology



- N is the number of channels, λ_F the Fermi wavelength
- l is the mean free path, determined by the disorder
- The natural macroscopic variable is $s = L/l$
- If $s \gg N$: one-dimensional system, localized. But if $s < N$, while $s \gg 1$: metallic behaviour. At low disorder, the conductance satisfies
  (i) Ohm's law: $\langle G \rangle \propto s^{-1}$
  (ii) Universal conductance fluctuations: $\mathrm{Var}(G) = 2/(15\beta)$

A microscopic model

- Hilbert space: $l^2(\mathbb{Z}) \otimes \mathbb{C}^N$
- Hamiltonian: $H = H_{bal} + \lambda V$, where

$$H_{bal} = -\Delta \otimes 1 + 1 \otimes H_\perp$$

$$V = \{V(x,z) : x \in \{1,\dots,L\}, z \in \{1,\dots,N\}\}$$

  $V(x,z)$ are i.i.d. random variables, with $\mathbb{E}V(x,z) = 0$, $\mathbb{E}V(x,z)^2 = 1$

- The mean free path l is of order λ^{-2}
- The model induces a $2N \times 2N$ transfer matrix $M^\lambda(L)$, which determines the transmission eigenvalues $T_i(L)$, $i = 1,\dots N$, and the conductivity $G(L) \propto \sum_i T_i(L)$ (Landauer-Büttiker)
- All of these quantities are random variables

A random matrix model

A macroscopic model of random transfer matrices $\mathcal{M}(s)$: the DMPK theory. Two central assumptions:

(i) Ito equation: $d\mathcal{M}(s) = d\mathcal{L}(s)\mathcal{M}(s)$, where $\mathcal{L}(s)$ is a Brownian motion (up to symmetries)

(ii) Equivalent channel assumption: The scattering for a short piece of wire, $d\mathcal{L}(s)$, is as chaotic as possible
Mathematically: the law of $d\mathcal{L}(s)$ is invariant under unitary transformations (mixing of the channels)

Consequences:

- The transmission eigenvalues $\mathcal{T}_i$ can be viewed as interacting diffusing particles (not Dyson's Brownian motion)
- Successfully describes universal conductance fluctuations

Result

On the microscopic side, let $L = \lambda^{-2} s$ and

- Choose $H_\perp$ to be an asymmetric hopping, with strength $h_\perp$:

$$H_\perp(z, z') = h_\perp \left(\mathrm{e}^{\mathrm{i}\gamma} \delta_{z,z'+1} + \mathrm{e}^{-\mathrm{i}\gamma} \delta_{z,z'-1} \right), \qquad z \in \mathbb{Z}/N\mathbb{Z}$$

- 'Interaction picture': $A^\lambda(L) := \big(M^0(L)\big)^{-1} \cdot M^\lambda(L)$

Theorem. [DMPK theory as a scaling limit of the Anderson model]
For all $s > 0$, the following equality holds in distribution:

$$\lim_{h_\perp \downarrow 0} \lim_{\lambda \downarrow 0} A^\lambda(\lfloor \lambda^{-2} s \rfloor) = \mathcal{M}(s/c),$$

where $c = c(E)$ is a constant of order 1.

Conjecture: The limit $h_\perp \downarrow 0$ can be replaced by $N \uparrow \infty$ 'in some sense'.